THE ALGEBRAICAL EQUATION AND PROBLEM PAPERS,

PROPOSED IN THE EXAMINATIONS

OF

ST. JOHN'S COLLEGE, CAMBRIDGE,

FROM THE YEAR 1794 TO THE PRESENT TIME,

WITH ANSWERS,

BY W. ROTHERHAM, B.A.

ST. JOHN'S COLLEGE.

CAMBRIDGE:

PRINTED FOR HENRY WALLIS, SIDNEY STREET;

WHITTAKER AND CO., AVE MARIA LANE;

AND G. BELL, 186, FLEET-STREET, LONDON.

M.DCCC.LII.

PREFACE.

In submitting to students this collection of Algebraical Equations and Problems, a short historical notice of the papers themselves may be acceptable, as affording an illustration of the rise and general progress of examinations in this college.

Our periodical examinations were first established by Dr. Powell, an able and zealous Master, about the year 1770. It is questionable whether the whole of the subjects were then introduced; but that Algebraical problems were included admits of no doubt, since there is one extant which is attributed to that Master. The problem is inserted in Bland, and commences thus,—"A silversmith received in payment for a certain weight of wrought plate," &c. From a very early period, these papers have appeared with great regularity in the May or June of each year; but I have been unable to discover whether they were printed before 1793. In that year however, a gentleman, who has favoured me with a communication, recollects perfectly well to have received a printed paper of quadratic equations. Examinations in the other subjects were conducted *vivâ voce* until the year 1808, when the questions were printed.

The first eleven papers have been obtained from a MS.; whose imperfections have compelled me to make substitutions in several places. The first three are without date, and probably are earlier than the date which, for the sake of preserving a consecutive order, I have assigned them.

Problems requiring the aid of probabilities appear in some of the papers. A remarkable instance occurs in the last problem of 1801. It runs thus in the MS.,—"*A* and *B*, whose skill is equal, play at bowls on this condition, that he who first gains eleven, shall receive a guinea; after some time *B* agrees to pay *A* 3 shillings and give up the game; but had each got one point more he must have given 3*s*. 6*d*. Quære, the state of the game." The impracticability of solving this by any elementary method (or of obtaining the *actual* answer, I apprehend, by any method) determined me to discard it. The discussion of the general problem will be found in a long investigation by Professor de Morgan, Theory of Probabilities.

A large number of equations and problems contained errors so cardinal as to be entirely useless; these I have remodelled, so as to preserve not only the principal, but also as far as the case has admitted, the subordinate features, and the original phraseology.

It is interesting to observe what a contrast in difficulty the earlier and later papers present; what little dexterity need be exhibited in the former, compared with what is

required in the latter, upon whose composition all the resources of modern ingenuity have been brought to bear. And these gradations of difficulty, it is hoped, will render the collection serviceable to incipient, as well as to more mature Algebraists.

I shall be glad to introduce into a future edition any earlier papers or suggested improvements with which I may be favoured. I may add that it is my intention to publish at an early opportunity, hints for the solutions of the more difficult equations and problems.

To the Reverend the Master of St. John's College, for a large contribution of early papers, I am under the deepest obligations. To the Rev. L. P. Baker, Rev. E. Simons, Rev. R. H. Newell, and other gentlemen, for copies of papers; and to other friends for their kind advice and assistance during the progress of the work, I owe my sincere acknowledgements.

W. R.

ST. JOHN'S COLLEGE,
CAMBRIDGE,
March 20, 1852.

EXAMINATION PAPERS.

1794.

1. $20 + \dfrac{7x^2}{3x} - 6 = \dfrac{352 - 12x}{10}.$

2. $\left.\begin{aligned} 4x - \frac{15 - x}{2} &= \frac{30y}{12} \\ 15x - 8y &= 35 - \frac{2x + 5y}{5} \end{aligned}\right\}$

3. $\left.\begin{aligned} x^2y^4 - 7xy^2 - 945 &= 765 \\ xy - y &= 12 \end{aligned}\right\}.$

4. A shepherd had two flocks of sheep, the smaller of which consisted entirely of ewes, each of which brought him 2 lambs. Upon counting them, he found that the number of lambs was equal to the difference between the two flocks, and that if all his sheep had been ewes and had brought him 3 lambs apiece, his stock would have been 432. Required the number in each flock.

5. A countryman, being employed by a poulterer to drive a flock of geese and turkeys to London, in order to distinguish his own from any he might meet on the road, pulled 3 feathers out of the tails of the turkeys, and 1 out of those of the geese, and upon counting them found that the number of turkey feathers exceeded twice those of the geese by 15. Having bought 10 geese and sold 15 turkeys by the way, he was surprised to find as he drove them into the poulterer's yard, that the number of geese exceeded the number of turkeys in the proportion of 7 : 3. Required the number of each.

6. Two persons, A and B, comparing their daily wages, found that the square of A's wages exceeded the square of B's by 5; and that if to the square of the sum of the fourth powers of their wages, there was added 4 times the rectangle contained by the square of the product of their wages and the square of the difference of the squares of their wages, augmented by 12 times the 4th power of the product of their wages, the aggregate amount would be £1428. 1*s*. Required the wages of each.

1795.

1. $$\frac{x-5}{4} + 6x = \frac{284 - x}{5}.$$

2. $$\left.\begin{aligned} \frac{x-2}{5} - \frac{10-x}{3} &= \frac{y-10}{2} \\ \frac{2y+4}{3} - \frac{2x+y}{8} &= \frac{x+13}{4} \end{aligned}\right\}$$

3. $$4x - \frac{x+3}{x-3} = 13 - x.$$

4. A detachment from an army was marching in a regular column, with 5 men more in depth than in front, but upon the enemy coming in sight, the front was increased by 845 men, and by this movement, the detachment was drawn up in 5 lines. Required the number of men.

5. £1000 was paid with pieces of coin, consisting of 36 shilling Portugal pieces, guineas and crowns; the number of Portugal pieces and crowns together is to the number of Portugal pieces and guineas together as 7 : 45; also the number of guineas and crowns together is to the number of guineas diminished by the number of Portugal pieces as 48 : 41. What was the whole number of pieces?

6. A certain company consists of men and boys, the number of men being greater; it is required to find the number

of both from the following data. If 8 times the number of men be added to 23 times the number of boys, the sum will be double of the sum of the cubes of the numbers; also, if to 34 times the number of boys, be added the difference between 6 times the square of the number of men and 5 times the square of the number of boys, the sum will be equal to 13 times the product of the number of men and boys increased by 24.

1796.

1. $\frac{3x}{4} - 4 = \frac{2x}{3} - 2.$

2. $\left.\begin{aligned} xy + 3y &= 20 \\ 5y - 4 &= 2xy \end{aligned}\right\}$

3. $5x - \frac{3x - 3}{x - 3} = 2x + \frac{3x - 6}{2}.$

4. A and B began to pay off their debts with different sums. A's money at first was $\frac{2}{3}$rds of B's: but after A had paid £1 less than $\frac{3}{4}$ths of his money and B £1 more than $\frac{7}{8}$ths of his, it was found that B had only half as much as A. What had each at first?

5. £500 was lent at simple interest in two separate sums, the smaller sum at 2 per cent. more than the other: the interest of the greater was afterwards increased and that of the smaller diminished 1 per cent. By this alteration, the whole interest was augmented one fourth of its former value; but if the interest of the greater had been so increased, without any diminution in that of the less, the interest of the whole would have been increased a third. Required the different sums and the rate of interest of each.

6. A square courtyard consists of a square grass plat (in the middle) and a gravel walk surrounding the grass plat. The side of the court wants 2 yards of being 6 times the breadth of the gravel walk; and the no. of square yards in the gravel walk exceeds the no. of yards in the periphery of the court by 92. Find the area of the court and the breadth of the gravel walk.

1797.

1. $7x - \dfrac{13 - 2x}{5} = 29 - \dfrac{x + 2}{3}$.

2. $\left.\begin{aligned} 4x - \dfrac{11y - \dfrac{y}{2}}{17 - 3x} &= 20 - \dfrac{103 - 8x}{2} \\ 8y + \dfrac{3y - 3}{5x - 10} &= 50 - \dfrac{147 - 24y}{3} \end{aligned}\right\}$

3. $\dfrac{123 + 41\sqrt{x}}{5\sqrt{x} - x} = \dfrac{20\sqrt{x} + 4x}{3 - \sqrt{x}} - \dfrac{2x^2}{(5\sqrt{x} - x)(3 - \sqrt{x})}$.

4. At an election, where each elector may give 2 votes to different candidates, but only one to the same, it is found on casting up the poll that of the 3 candidates, A, B, C, A has 158 votes, B 132, and C 118. Now 26 electors voted for A alone, 30 for B alone, and 28 for C alone. How many voted for A and B jointly? how many for A and C? and how many for B and C?

5. Round two wheels, whose circumferences are as 5:3, two ropes are wrapped, the difference of the ropes exceeding the difference of the circumferences of the wheels by 280 yards. Now the rope applied to the larger wheel wraps round it a number of times greater by 12 than the number of times which the other rope goes round the other wheel, and if the larger turns round 3 times for the other once, the ropes will be disengaged from each in the same time. Required the lengths of the ropes and the circumferences of the wheels.

6. The sum of 3 guineas is to be raised by 2 estates, which are to be charged in proportion to their values. Of this sum, *A*'s estate, which is 4 acres more than *B*'s, but worse by 2*s*. an acre, pays £1. 15*s*. But had it possessed 6 acres more, and *B*'s land had been worth 3*s*. an acre less than before, every thing else remaining the same, *A* would have paid £2. 5*s*. Required the number of acres in each estate.

1798.

1. $\dfrac{x+2}{5} + \dfrac{3x-1}{4} = 5 - \dfrac{5x+1}{8}.$

2. $$\left.\begin{aligned} 2 - \frac{\frac{5x}{y} - 3}{4} &= 7 - \frac{4x+3}{2y} \\ 11y + \frac{6 - \frac{y}{3}}{5} &= 35 - \frac{\frac{x}{5} + 4}{7} \end{aligned}\right\}$$

3. $\dfrac{1}{x^2 - 3x} + \dfrac{1}{x^2 + 4x} = \dfrac{9}{8x}.$

4. $$\left.\begin{aligned} x^4 - 2x^2y^2 + y^4 - x^2 + y^2 &= 20 \\ x^4 - 2x^2y + y^2 &= 49 \end{aligned}\right\}$$

5. A regiment of militia has just men sufficient to form an equilateral wedge; it is afterwards doubled by the supplementary, but it is still found to want 385 men that it may complete a square containing 5 men more in a side than the wedge had. Required the number of men in the regiment.

6. In one of the corners of a rectangular garden there is a fish-pond, whose area is a ninth part of the whole garden. The periphery of the garden exceeds that of the fish-pond by 200 yards. Also if the greater side be increased by 3 yards and the other by 5 yards, the garden will be enlarged 645 square yards. *N.B.* The fish-pond is a rectangle about the same diameter with the garden. Required the periphery of the garden.

7. Four persons *A, B, C,* and *D,* undertake to reap a field of 25 acres. The four terms of an Arith. Prog., whose sum is 20, will express the times in which they can severally reap an acre. Moreover, they altogether can finish the undertaking in 24 days. In how many days can *A, B, C,* and *D* alone reap an acre?

1799.

1. $\dfrac{3x}{5} + \dfrac{x}{10} - 4 = \dfrac{34 - 7x}{2}$.

2. $$\left.\begin{aligned} \frac{4y}{5} - \frac{3 - y}{8} &= \frac{y + \frac{139}{7}}{15} \\ \frac{77y - 139}{2} &= \frac{135 - 15y}{2x} \end{aligned}\right\}$$

3. $\dfrac{10}{x} - \dfrac{14 - 2x}{x^2} = 2\frac{4}{9}$.

4. $$\left.\begin{aligned} x^2 + \frac{3xy}{4y^2} &= 10\frac{1}{8} \\ xy &= \frac{y}{2} + 5 \end{aligned}\right\}$$

5. A field of wheat and oats, which contained 20 acres, was put out to a labourer to reap for 6 guineas, the wheat at 7*s.* an acre, the oats at 5*s.*; but the labourer falling ill, reaped only the wheat. How much, if he is paid proportionally to the bargain, will he receive?

6. Some smugglers discovered a cave, which would exactly hold the cargo of their boat, viz:—13 bales of cotton, and 33 casks of rum, and then they resolved to unlade the whole. But a custom-house cutter coming in sight, they sailed away with 5 of the bales and 9 of the casks, leaving the cave two-thirds full. How many bales or casks would it hold?

7. An upholsterer has two square carpets, divided into square yards by the lines of the pattern. Now he observes, that if he subtracts from the number of squares in the smaller carpet the number of yards in the side of the larger, the square of the remainder will exceed the difference of the number of squares in the smaller carpet and the number of yards in its side by 88. Moreover, the difference of the lengths of the sides of the two carpets is 6 feet. Required the size of each.

1800.

1. $$6 - \frac{25 - x}{5} = \frac{x^2}{5x - 20}.$$

2. $$\left.\begin{aligned} 8x - \frac{16 + 60x}{3y - 1} &= \frac{16xy - 107}{5 + 2y} \\ 2 + 6y + 9x &= \frac{27x^2 - 12y^2 + 38}{3x - 2y + 1} \end{aligned}\right\}$$

3. $$16 - \frac{2x^2}{3} = \frac{4x}{5} + (7\tfrac{3}{5}) \times \frac{x}{3}.$$

4. $$\frac{33\frac{24}{25}}{\sqrt{5x^2 - x^4}} + \frac{\sqrt{5 - x^2}}{25x} = \frac{34}{x}.$$

5. Out of a pack of cards, a certain number, including the ten of diamonds, is dealt equally among 4 persons, the dealer turning up the last card that he gives himself, which is the ten of spades. Now if twice the number had been dealt to each, the ten of diamonds being still dealt out, and the dealer still turning up the ten of spades for his last card; the chance in favour of his having the ten of diamonds would have been to the chance against him as 3 : 10. Quære, what number of cards was dealt to each?

6. Before noon, a clock, which is too fast, and points to afternoon time, is put back 5 hours 40 minutes to true time, when it is observed that the time before shown by the clock is to the true time as 29 : 105. What time did the clock shew before it was put back?

7. There are two rows of cards, the upper row containing one more than the lower; a certain number of cards N which it is required to find, is taken from the upper row; and it is observed, that if as many be taken from the lower row as remain in the upper, the sum of the square of the number remaining in the lower row, and of its square root, equals a fraction whose numerator is 72 and denominator that number which is less than N by 1.

1801.

1. $\dfrac{x - ax}{\sqrt{x}} = \dfrac{\sqrt{x}}{x}.$

2. $\left.\begin{array}{l}\dfrac{2y}{18} - \dfrac{8x - 2}{36} = 1 - \dfrac{4 + y}{3} + \dfrac{x - 7}{6} \\ x : 3y :: 4 : 7\end{array}\right\}$

3. $\dfrac{7 - 12x}{x^{\frac{3}{2}}} = \dfrac{x}{\sqrt{x}} - \dfrac{8x + 110}{x^{\frac{3}{2}}}.$

4. $\dfrac{49x^2}{4} + \dfrac{48}{x^2} - 49 = 9 + \dfrac{6}{x}.$

5. A certain sum of money is divided every week among the resident fellows of St. John's. It happened one week that the number resident was the square root of the number of £s to be divided. Two men, however, coming into residence the week after, diminished the dividend of each of the former individuals £1. 6*s*. 8*d*. What was the sum to be divided?

6. A city barge, with chairs for the company and benches for the rowers, went a summer excursion, with two bargemen on every bench. The no. of gentlemen on board was equal to the square of the no. of bargemen, and the no. of ladies was equal to the no. of gentlemen, twice the no. of bargemen and one over. Among other provisions, there were a no. of turtles equal to the square root of the no. of ladies; and a no. of bottles of wine less than the cube of the no. of turtles by 361. The turtles in dressing consumed a great quantity of wine, and the party having staid out till the turtles were all eaten, and the wine all gone, it was computed, that supposing them all to have consumed an equal quantity, (viz. gentlemen, ladies, bargemen, and turtles) each individual would have consumed as many bottles as there were benches in the barge. Required the no. of turtles.

7. *A* and *B* playing at bowls, says *A* to *B*, if you will give me a guinea, I will bet you half-a-crown to eighteenpence on each game, and will play 36 games together. *B* won his guinea back again, and £1. 17*s*. besides. How many games did each win?

1802.

1. $7x - \dfrac{13 - 2x}{5} = 29 - \dfrac{x + 2}{3}.$

2. $$\left.\begin{aligned} x - \frac{2y - x}{23 - x} &= 20 - \frac{59 - 2x}{2} \\ y + \frac{y - 3}{x - 18} &= 30 - \frac{73 - 3y}{3} \end{aligned}\right\}$$

3. $$\frac{x^2 - \dfrac{5x - 21}{2}}{x} = 6$$

4. $$\frac{\sqrt{x^2 + x + 6}}{3} = \frac{18 - (\frac{4}{3}\sqrt{x^2 + x + 6} - 2)}{\sqrt{x^2 + x + 6}}.$$

5. From each of 16 coins, an artist filed the worth of half-a-crown, and then offered them in payment for their original value; but being detected, the pieces were found to be really worth no more than 8 guineas. What was their original value?

6. The fore wheel of a carriage makes 6 revolutions more than the hind wheel in going 120 yards; but if the periphery of each wheel be increased 1 yard, it will make only 4 revolutions more than the hind wheel in the same space. Required the number of yards in the periphery of each wheel.

7. In *A*'s garden is a square bowling green, a side of which is 30 yards and near to it is a rectangular grass plat. Now the number of square yards in the area of the grass plat is a mean proportional between $\frac{192}{79}$ and the square yards in the grass plat and bowling green together. Also the number of square yards contained in the square described on the diameter of the grass plat is a mean proportional between 10 and the number of square yards contained in the aforesaid square, increased by the number contained in the bowling green. Quære, the area and sides of the grass plat.

1803.

1. $3x + \dfrac{2x+1}{5} = 4 + \dfrac{11x - 37}{2}$.

2. $\left.\begin{array}{l} x - \dfrac{3y - 2 + x}{11} = 1 + \dfrac{15x + \dfrac{4y}{3}}{33} \\ \dfrac{3x+y}{6} - \dfrac{y-5}{4} = \dfrac{11x + 152}{12} - \dfrac{3y+1}{2} \end{array}\right\}$

3. $3x - \dfrac{1121 - 4x}{x} = 2.$

4. $\left.\begin{array}{l} y^2 - 64 = 8x^{\frac{1}{2}}y \\ y - 4 = 2x^{\frac{1}{2}}y^{\frac{1}{2}} \end{array}\right\}$

5. A landlord let his farm for £10 a year in money and a corn rent: when corn sold at 5*s.* a bushel, he received at the rate of 10*s.* an acre for his land, but when it sold at 7*s.* 6*d.* a bushel, 13*s.* an acre. Of how many bushels did the corn rent consist?

6. *A* and *B* each bought £300 in the stocks; *A* into the 3 per cents. and *B* into the fours. These stocks were at such a price that *B* received £1 interest more than *A*. When afterwards each of the stocks rose £10 per cent. they sold out their money and *A* found himself £10 richer than *B*. Required the price of stock.

7. A company of merchants fitted out a privateer, each subscribing £100. The captain subscribed nothing, but was entitled to a £100 share at the end of every certain no. of months. In the course of 25 months, he captured 3 prizes, the values of which were in Geom. Prog. the middle term being one fourth of the cost of the equipment, the common ratio being the no. of months in which the captain was entitled to his £100 share, and the sum of 3 terms £1375 more than the cost of the equipment. After deducting £875 for prize money to the crew, the captain's share of the remainder amounted to one fifth of that of the company. Required the no. of merchants and the pay of the captain.

1804.

1. $\dfrac{4x-21}{9}+3\tfrac{3}{4}+\dfrac{57-3x}{4}=241-\dfrac{5x-96}{12}-11x.$

2. $\left.\begin{aligned}\dfrac{80+3x}{15}&=18\tfrac{1}{3}-\dfrac{4x+3y-8}{7}\\ 10y+\dfrac{6x-35}{5}&=55+10x\end{aligned}\right\}.$

3. $3x-\dfrac{3x-10}{9-2x}=2+\dfrac{6x^2-40}{2x-1}.$

4. $\left.\begin{aligned}y+\sqrt{\dfrac{y}{x}}&=\dfrac{42}{x}\\ \dfrac{x^3}{3}+\dfrac{x}{2\sqrt{y}}&=\dfrac{54}{y}\end{aligned}\right\}.$

5. A body of men were formed into a hollow square, three deep, when it was observed that with the addition of 25 to their no., a solid square might be formed of which the no. of men in each side would be greater by 32 than the square root of the no. of men in each side of the hollow square. Required the no. of men in the hollow square.

6. Two Spanish muleteers, *A* and *B*, were seated under a tree to dine; and on examining, found their stock of provisions to consist of 5 small loaves of bread, three of which were *A*'s property, and a bottle of wine, which was *B*'s. A stranger who happened to come up at the time, was invited to partake of the fare, which was just sufficient for 3 persons; and at parting, being pleased with their behaviour, he gave them what Spanish money he had about him, which amounted to 6*s*. 5½*d*. to be equitably shared between them. Now as many shillings as a loaf cost pence would, with fourpence more, at the next town have bought six such loaves and four bottles of the same wine; and when the money was divided, *B* received 1*s*. 10½*d*. more than *A*. What was the price of each loaf and a bottle of wine?

7. A company at a tavern had £8 15*s*. to pay; but before the bill was paid, two sneaked off, when the remainder had each 10*s*. more to pay. Required the no. at first.

1805.

1. $\frac{x-1}{7} + \frac{23-x}{5} = 7 - \frac{4+x}{4}.$

2. $$\left.\begin{aligned} 3 - \frac{7 + \frac{2x}{y}}{5} &= 5 - \frac{5x+9}{3y} \\ y - \frac{4+15y}{6x-2} &= \frac{2xy - \frac{107}{8}}{2x+5} \end{aligned}\right\}$$

3. $2x^2 + 3x - 5\sqrt{2x^2 + 3x + 9} + 3 = 0.$

4. $$\left.\begin{aligned} \sqrt{5\sqrt{x} + 5\sqrt{y}} + \sqrt{y} &= 10 - \sqrt{x} \\ \sqrt{x^5} + \sqrt{y^5} &= 275 \end{aligned}\right\}$$

5. A man has two cubical vessels and the side of the greater exceeds that of the less by 2 feet. Also the greater will contain 218 solid feet of water (about 1336 gallons) more than the less. Required the dimensions of each.

6. One buys 2 sorts of cloth for £7 18*s*. the no. of shillings given for a yard of the better = the no. of yards in both pieces, the no. of shillings given for a yard of the worse = the excess of the no. of yards in the better piece above the no. in the worse and the whole price of the best piece : whole price of worse :: 72 : 7. Quære, the no. of yards?

7. A no. of shillings being placed at equal distances on a table so as to form an equilateral triangle, from the middle of each side as many shillings as equal the square root of the no. of shillings on that side were taken and placed on the corner shilling at the opposite angle; after which it appeared, that the no. of shillings now in any one side is to the no. before on that side as 5 to 4. Required the no. on each side.

1806.

1. $x - \dfrac{33 - x}{x} = \dfrac{36x - 99}{3x}$.

2. $\left.\begin{array}{l}(x + 5)(y + 7) = (x + 1)(y - 9) + 112. \\ 2x + 10 = 3y + 1\end{array}\right\}$

3. $\dfrac{nx + b}{\sqrt{x}} = \dfrac{na + b}{\sqrt{a}}$.

4. $\left.\begin{array}{r}x^3 - y^3 : (x - y)^3 :: 61 : 1 \\ xy = 320\end{array}\right\}$

5. There are two casks A and B; of which A the greater holds 312 gallons. Into A a certain quantity of wine is put, and B is filled with water. Now the water is conveyed out of B into A at different times in the following manner. First, a no. of gallons is taken which is less by 2 than the square root of the gallons of wine in A. Then a quantity less than the former by 2 gallons and so on. Now when B is in this manner exactly emptied, it is found that A is exactly full, and it is known that 8 gallons were taken out of B at one time, after which the quantity left in B was 12 gallons. Quære, the no. of gallons of wine in A.

6. Suppose that out of a cask of wine holding 81 gallons when full, a certain quantity was drawn and the rest filled up with water, and that the same quantity of the mixture was afterwards drawn and supplied by water three several times; and that then it appeared, that besides water there were but 16 gallons left in the cask. How much was drawn at each time?

7. A and B undertake to reap a field of corn in 5 days for £4 10*s.*, but were obliged to call in C to assist them for the last two days, by which means B alone lost 3*s.* 9*d.* How long would B or C be in doing the work alone, if A could do it in 9 days?

1807.

1. $\dfrac{5x+1}{4} - \dfrac{7-7x}{6} = 16.$

2. $$\left.\begin{array}{l} x - \dfrac{y+12}{5} : y - \dfrac{24-5x}{8} :: 1 : 5 \\[2ex] \dfrac{x-2}{2} - \dfrac{19-y}{3} = \dfrac{\frac{x}{2}+8}{6} - \dfrac{y-9}{4} \end{array}\right\}$$

3. $5x - \dfrac{37-3x}{14-x} = 43.$

4. $5x^2\sqrt[3]{\dfrac{1}{x^2}} + 2x\sqrt[3]{\dfrac{1}{x}} = 16 - 9x\sqrt[3]{x}.$

5. When flour cost 7*s.* a bushel, the baker sold a loaf for 16*d.*; and when it rose to 10*s.* 6*d.* he sold a loaf of the same weight for 21*d.*: the price of baking being the same in each case. Required that price.

6. In changing a bill of £85 into guineas and shillings, the no. of shillings being $\frac{1}{4}$ the no. of guineas, on examination, they all proved adulterated below the standard value to the amount, in the whole, of £8 5*s.* To make up the deficiency, 9 more such guineas were paid; and 4 such shillings and 3 good ones, returned. Required the no. and average value of the guineas and shillings paid at first.

7. *A* and *B* engaged to reap a field of corn in 12 days. The times in which they could severally reap an acre, are to one another in the proportion of 2 : 3. After some time, finding themselves unable to finish it in the stipulated time, they called in *C* to help them: whose rate of working was such, that had he wrought with them from the beginning, it would have been finished in 9 days. Also the times in which he could severally have reaped the field with *A* alone and *B* alone are to one another in the proportion of 7 : 8. When was *C* called in?

1808.

1. $\dfrac{x-3}{2} - \dfrac{x+2}{3} = \dfrac{8-x}{4}.$

2. $$\left.\begin{array}{l} x + y : 4x + y :: 4 : 7 \\ \dfrac{\dfrac{11y}{6} - 2x}{5} - \dfrac{21-3y}{4} = \dfrac{2 + \dfrac{x}{10}}{3} - \dfrac{1}{12} \end{array}\right\}$$

3. $16 - \dfrac{5-x}{2} = \dfrac{9+3x}{x} + 3x.$

4. $\dfrac{x^2+8}{6} - \dfrac{4x+6}{x^2} = \dfrac{8 - \dfrac{2}{x^2}}{3}.$

5. A flock of ewes, of which $\frac{1}{10}$ were barren, and $\frac{1}{4}$ brought twins, produced 23 lambs. Required the number of ewes, none being supposed to produce more than 2.

6. The captain of a privateer descrying a trading vessel 7 miles a-head, sailed 20 miles in direct pursuit of her, and then observing the trader turn short to the right, changed his own course, so as to overtake her without making another tack. On comparing their reckonings it was found, that the privateer had run at the rate of 10 knots in an hour, and the trading vessel at the rate of 8 knots in the same time. Required the distance sailed by the privateer.

7. A rectangular vat, 3 feet deep, when filled to the depth of 2 feet, holds less than when completely full, by a no. of cubic feet equal to 24 together with half the no. of feet in the perimeter of the base. It is also observed, that the length of a pole which reaches from one of the corners of the top to the opposite corner of the bottom of the vat, is equal to one eighth of the no. of feet in the square inscribed on the diagonal of the bottom. Required the dimensions of the vat.

1809.

1. $x + \frac{x+1}{3} = \frac{x-2}{6} + 10.$

2. $\left.\begin{array}{l} 2.4\,x + .32\,y - \frac{.36\,x - .05}{.5} = .8x + \frac{2.6 + .005\,y}{.25} \\ \frac{.04\,y + .1}{.3} = \frac{.07\,x - .1}{.6} \end{array}\right\}$

3. $\left.\begin{array}{l} (2x - 4y)^2 + x - 2y = 5. \\ x^2 - y^2 = 8. \end{array}\right\}$

4. $x^2 - 2x^{\frac{3}{2}} + 2x - \sqrt{x} = 6.$

5. Water flows uniformly into a cistern, capable of containing 720 gallons, through a pipe; and at the same time is discharged by a pump, worked by 3 men, who take 4 strokes in a minute; but this not being sufficient, the cistern becomes full in 6 hours; they therefore now put in another pump, of such power that the quantity discharged at one stroke by this pump is to the quantity discharged at one stroke by the former :: 2 : 3; but being obliged to detach one of their number to work this pump, the former pump makes only 10 strokes in 3 minutes, and the latter 5 strokes in 2 minutes; by which means the cistern is emptied in 12 hours. How much water was discharged by each pump at one stroke; and how much flowed in through the pipe in one minute?

6. When a parish was inclosed, the allotment of one of the proprietors consisted of two pieces of ground; one of which was in the form of a right-angled triangle; the other was a rectangle, one of the sides of which was equal to the hypothenuse of the triangle, the other to half the greater side; but wishing to have his land in one piece, he exchanges his allotments for a square piece of ground of equal area, one side of which equalled the greater of the sides of the triangle which contain the right angle. By this exchange he finds that he has saved ten poles of

railing. What are the respective areas of the triangle and rectangle; and what is the length of each of their sides?

7. A pyramidic pile of cannon balls, the base of which was an equilateral triangle, was all used in an engagement, except the three lowest layers and 4 balls of the next layer; these were afterwards formed into a pile with a rectangular base, having as many balls in one side of the lowest layer as there were in a side of the lowest layer of the pyramidic pile, and four in the adjacent side. What was the number of balls; and what the number of layers in each pile, when complete?

1810.

1. $\dfrac{4x+3}{3} - 18 = \dfrac{x-3}{6} - \dfrac{3x-15}{2}.$

2. $\left.\begin{aligned} &\dfrac{4y-x}{7} - \dfrac{12x-14}{5} = 1 - \dfrac{64y+11x-3}{35} \\ &\dfrac{7x - \frac{4}{3}y}{3} = \dfrac{2y-1}{2} + \dfrac{30-2x}{4} \end{aligned}\right\}$

3. $\dfrac{2x-1}{3-x} = \dfrac{8-x^2}{2x-2} + \dfrac{x}{2}.$

4. $\left.\begin{aligned} &\sqrt{y} + \sqrt{x} : \sqrt{y} - \sqrt{x} :: \sqrt{x} + 2 : 1 \\ &\dfrac{\sqrt{y}+2}{\sqrt{x}} - 1 = \dfrac{3\sqrt{x} + 1 + \dfrac{\sqrt{y}}{\sqrt{x}}}{\sqrt{y}} \end{aligned}\right\}$

5. Two merchants speculate with different sums, *A* with £200 and *B* with a sum unknown. *A* gained 35 per cent. by his speculation, and *B* lost 60 per cent. by his: after which the sum of their stocks was found to be equal to the sum of *B*'s loss and *A*'s gain. Required *B*'s stock at first.

6. On the late jubilee, a gentleman treated his tenantry at the following rate. He allowed for each poor child a certain

no. of sixpences, for each poor woman sixpence more, and for each poor man sixpence still in addition. The no. of women was one-fourth greater than the no. of men; the no. of children was equal to twice the square of the difference between the nos. of men and women; and the whole expense was £8 2*s*. But had each child been allowed as much as each woman, the expense on this account added to nine times the difference of what the men and women cost, would have been £4 18*s*. Required the no. of men, women, and children, and the allotment to each.

7. The diagonals of four squares are in an increasing Geom. Prog. and the product of the squares of the diagonals of the extremes : the product of the diagonals of the means :: a side of the third : square root of the common ratio divided by $4\sqrt{2}$. Required the diagonal of the third square, and the common ratio, supposing their difference equal to 45.

1811.

1. $$\frac{75}{8} + \frac{3x - 1}{4} - \frac{7x + 3}{16} = \frac{8x + 19}{8}.$$

2. $$\left.\begin{aligned} &\frac{\frac{7x}{4} + 6y}{5} - \frac{\frac{3y + 6}{5} - \frac{3x - 2}{10}}{8} = 5 - \frac{x}{16} \\ &\frac{3x}{2} + \frac{2y}{3} + 2\tfrac{1}{2} : \frac{x}{2} - \frac{y}{3} + \frac{1}{6} :: 10\tfrac{1}{2} : 1\tfrac{1}{6} \end{aligned}\right\}$$

3. $$\frac{x}{x + 9} + \frac{5}{2x + 18} = \frac{x - \frac{x^2 + 20}{x + 8}}{2}.$$

4. $$\left.\begin{aligned} &\frac{2y^2 - 8\sqrt{x}}{\sqrt{x}} + \sqrt{4y^2 - 16\sqrt{x}} = \frac{3\sqrt{x}}{2} \\ &\sqrt{x} + \sqrt{8(y - \sqrt{x}) - 4} = y + 1 \end{aligned}\right\}$$

5. 400 acres are equally divided amongst a certain no. of

farmers; one of them, by adding 5 acres to his stock, increases his property in the ratio of 5 : 4. Required the no. of farmers.

6. A body of men are just sufficient to form a hollow equilateral wedge, 3 deep; and if 597 men be taken away, the remainder will form a hollow square, 4 deep, the front of which contains one man more than the square root of the no. contained in a front of the wedge. Required the no. of the men.

7. The Fly starts 10 miles before the Telegraph; but the Fly coachman having made an appointment with the driver of the Telegraph, walks his horses so as to be overtaken at the end of the second mile. Now it is observed, that the no. of revolutions made in a given time by the hinder wheel of the Fly, its forewheel, and the hinder wheel of the Telegraph, increase in Arith. Prog., and that the circumferences of *these* wheels, viz. of the forewheel of the Fly, its hinder wheel, and the hinder wheel of the Telegraph, increase in Geom. Prog., whose common ratio is the same as the common difference of the Arith. Prog. It is required to find the ratio which the wheels bear to each other.

1812.

1. $\dfrac{x+1}{3} + \dfrac{4x+7}{9} = 4 + \dfrac{x}{5}.$

2. $\left.\begin{array}{l} \dfrac{2x+y}{9} + \dfrac{7y+6x+11}{18} = 9\frac{1}{2} - \dfrac{5x-17}{6} \\ \dfrac{5x+3y+2}{7} : \dfrac{9y+6}{2} :: 1 : 3 \end{array}\right\}$

3. $\dfrac{2x^2}{3} + 3\frac{1}{2} - \dfrac{x}{2} + 8.$

4. $\left.\begin{array}{l} \sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}} = \dfrac{61}{\sqrt{xy}} + 1 \\ \sqrt[4]{x^3y} + \sqrt[4]{y^3x} = 78 \end{array}\right\}$

5. A fruiterer sold for 19*s*. 6*d*. a certain no. of oranges and apples, of which the latter exceeded the former by 180. He sells the apples at the rate of five for 3*d*., and fifteen oranges bring him in $1\frac{1}{2}d$. more than 35 apples. How many are there of each sort, and what are the oranges worth apiece?

6. Last winter, when fuel was very scarce, two men, *A* and *B*, went in quest of coals and turf, which they agreed to use in common. *A* met with three bushels of coals, and *B* two, at the same price per bushel, and also seven baskets of turf. *A* stipulated that he should consume twice as many coals as *B*. *B* assented, but demanded of him 2*s*. 10*d*. When this stock was exhausted, *B* purchased one bushel of coals, and *A* five, together with 6 baskets of turf, at the same rates respectively as before; but now *B* consumed three times as many coals as *A*, and paid him 18*s*. 6*d*. What was the price of a bushel of coals, and of a basket of turf; equal quantities of turf having been consumed by each person?

7. A number (n) of boys arrange themselves in a right line at equal intervals, the nearest being at a given distance (a) from a fixed station S. A person P walks from S to the first boy, and as soon as he begins to return, the remaining boys move *from* S at the same rate as P, and stop when he comes to S. P advances again to the second boy, and whilst he is returning, the remainder move onward, and halt as before. The same is repeated until P has reached the last boy and returned. Now if under the same circumstances the boys had moved each time towards S, P would have passed over only $\frac{1}{m}$th part of his former distance. Shew that the distance (d) of the boys from each other is equal to

$$\frac{(2^n - m - 1)\,a}{n(m+1) - (2^n + m - 1)}$$

1813.

1. $x - \dfrac{x+1}{5} - \dfrac{2x+3}{3}.$

2. $$\left.\begin{array}{l} \dfrac{3x+2y}{5} - \dfrac{5x - \dfrac{3y}{4} + 1}{3} = x + \dfrac{y-2x}{10} - \dfrac{4x-y}{7} \\ y + 2x : y - 2x :: 12x + 6y - 3 : 6y - 12x - 1 \end{array}\right\}$$

3. $\dfrac{7+x}{7-x} + \dfrac{7-x}{7+x} = \dfrac{29}{10}.$

4. $$\left.\begin{array}{l} \sqrt{\dfrac{x+y^2}{4x}} + \dfrac{y}{\sqrt{y^2+x}} = \dfrac{y^2}{4}\sqrt{\dfrac{4x}{y^2+x}} \\ \dfrac{\sqrt{x} + \sqrt{x-y-1}}{\sqrt{x} - \sqrt{x-y-1}} = y + 1 \end{array}\right\}$$

5. The colonel of a regiment at Derby sent a detachment to Leeds to protect the machinery. The detachment was a third of the regiment; but the commanding officer at Leeds thinking his numbers insufficient, and suspecting that 50 of his men had been corrupted by the Luddites, ordered the suspected men back to Derby, with a request that the colonel would let him have half the regiment. To comply with his wishes, the colonel had every third man (of the remainder at Derby) drafted off and sent to Leeds. Of how many did the regiment consist?

6. *A, B* and *C* undertake a piece of work jointly, but after a certain no. of days *A* falls ill, and when *B* and *C* have completed the work, they receive respectively £4, £21, and £35. Had *B* fallen ill when *A* did, and *A* and *C* completed the work, it would have required 5 more days to have finished it. But, had *C* fallen ill instead of *A*, it would have taken 21 more days. Required the no. of days during which *A* worked, and the no. in which the work was completed.

7. The roof of a storehouse is formed of two squares terminated by two equal and parallel isosceles triangles. The

height of the walls equals the base of either of these triangles. The quantity of wood which the storehouse will hold, increased by 6 cubical piles, each of the same length as the building : quantity which the same storehouse would hold if its roof were flat :: 11 : 2. The roof cost as many pence per square foot, as there are feet in its ridge, and the flooring was laid at the same rate. Both together cost £208 6*s*. 8*d*. Required the dimensions of the storehouse.

1814.

1. $$\frac{3x+7}{14} - \frac{2x-7}{21} + 2\tfrac{3}{4} = \frac{x-4}{4}.$$

2. $$\left.\begin{aligned} 3x + 6y + 1 &= \frac{6x^2 + 130 - 24y^2}{2x - 4y + 3} \\ 3x - \frac{151 - 16x}{4y - 1} &= \frac{9xy - 110}{3y - 4} \end{aligned}\right\}$$

3. $$\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\tfrac{1}{3}.$$

4. $$\left.\begin{aligned} 5y + \frac{\sqrt{x^2 - 15y - 14}}{5} &= \frac{x^2}{3} - 36 \\ \frac{x^2}{8y} + \frac{2x}{3} &= \sqrt{\frac{x^3}{3y} + \frac{x^2}{4}} - \frac{y}{2} \end{aligned}\right\}$$

5. A brewer, from a certain quantity of ingredients which cost £20, brews 500 gallons of ale (on which there is a duty of 6*d*. a gallon), and sells it at 2*s*. a gallon. Afterwards, from the same quantity of ingredients, he brews a certain no. of gallons of strong beer (on which he pays the ale duty), and the remainder small beer, making together the same no. of gallons as before,—when, by mixing them together, and selling the mixture as ale, he finds his gains increased in the proportion of 10 : 7. Determine the no. of gallons of strong beer, supposing the duty on small beer $\frac{1}{4}$th of that on ale.

6. The no. of deaths in a besieged garrison amounted to 6 daily, and allowing for this diminution, their stock of provisions was sufficient to last for 8 days. But, on the evening of the sixth day, 100 men were killed in a sally, and afterwards the mortality increased to 10 daily. Supposing the stock of provisions unconsumed at the end of the 6th day sufficient to support 6 men for 61 days; it is required to find how long it would support the garrison, and the no. of men alive when the provisions were exhausted.

7. A man buys a guinea at the market price of standard gold, but an act of parliament passing which makes it illegal to sell the guinea in the same way that he bought it, he privately clips off one twenty-fifth part. He may now legally sell it as a light guinea, and he finds that in consequence of the rise of pure gold in the ratio of 239 : 249 he just gains the clippings by his purchase. It is required to find the ratio of pure gold and alloy in the guinea, and also the relative value of equal quantities of pure gold and alloy, it being known that the sum of the squares of the nos. which express the two ratios, exceeds eleven times their sum by $233\frac{9}{121}$.

1815.

1. $$\frac{2x+1}{29} - \frac{402-3x}{12} = 9 - \frac{471-6x}{2}.$$

2. $$\left.\begin{aligned} &\frac{3x-5y}{3} - \frac{2x-8y-9}{12} = \frac{y}{2} + \frac{1}{3} + \frac{1}{4} \\ &\frac{x}{7} + \frac{y}{4} + \frac{4}{3} : 4x - \frac{y}{8} - 24 :: 3\tfrac{1}{3} : 3\tfrac{1}{2} \end{aligned}\right\}$$

3. $$\frac{x+3}{2} + \frac{16-2x}{2x-5} = 5\tfrac{1}{5}.$$

4. $$\left.\begin{aligned} &\frac{x+y+\sqrt{x^2-y^2}}{x+y-\sqrt{x^2-y^2}} = \frac{9}{8y}(x+y) \\ &(x+y)^2 + x - y = 2x(x^2+y) + 506 \end{aligned}\right\}$$

5. At the review of an army, the troops were drawn up in a solid mass, 40 deep; when there were just one-fourth as many men in front as there were spectators. Had the depth, however, been increased by 5, and the spectators drawn up in a mass with the army, the no. of men in front would have been 100 fewer than before. Determine the no. of men of which the army consisted.

6. A no. of persons purchased a field for £345. The youngest contributed a certain sum, the next £5 more, the third £5 more than the second, and so on to the oldest. For the greater accommodation of the seniors, the field was divided into two parts, the younger half taking a portion proportional to the sum they had subscribed; and in order that each might have an equal share in this portion, they agreed to equalize their contributions, and each to pay £22. Required the no. of persons, and the sums paid by each.

7. *A* and *B* travelled on the same road, and at the same rate, from Huntingdon to London. At the 50th mile-stone from London, *A* overtook a drove of geese, which were proceeding at the rate of three miles in two hours; and two hours afterwards met a stage waggon, which was moving at the rate of nine miles in four hours. *B* overtook the same drove of geese at the 45th mile-stone, and met the same stage waggon exactly forty minutes before he came to the 31st mile-stone. Where was *B* when *A* reached London?

1816.

1. $$\frac{4x-34}{17}-\frac{258-5x}{3}=\frac{69-x}{2}.$$

2. $$\left.\begin{aligned} &\frac{4x-2y+3}{3}-\frac{18-x+5y}{7}=\frac{x}{4}-\frac{y}{5}-\frac{1}{7}-7\tfrac{1}{10} \\ &2x-y+15 : y-2x+15 :: \frac{x}{3}-\frac{y}{4}+\frac{3}{4} : \frac{y}{4}-\frac{x}{3}+\frac{1}{12} \end{aligned}\right\}$$

3. $\dfrac{2x}{x-4} + \dfrac{2x-5}{x-3} = 8\frac{1}{3}$.

4. $$\left.\begin{aligned} \frac{y}{x} - \frac{9\sqrt{x}}{y} - \frac{81}{xy} &= (2y+9)\frac{\sqrt{x}}{y} \\ \frac{\sqrt{y}}{x} + 3\sqrt{\frac{x}{y}} &= \frac{9}{x\sqrt{y}} + \sqrt{x} \end{aligned}\right\}$$

5. A packet sailing from Dover with a fair wind, arrives at Calais in two hours; and on its return the wind being contrary, it proceeds 6 miles an hour slower than it went. Now when it is half way over, the wind changing, it sails two miles an hour faster, and reaches Dover sooner than it would have done had the wind not changed, in the proportion of 6 : 7. Required the rates of sailing, and the distance between Dover and Calais.

6. From the middle of a town two streets branched off, and crossed a river that ran in a straight course, by two bridges *A* and *B*. From their junction, a sewer equally inclined to both streets led to a point in the river, at the distance of 6 chains from the bridge *A*, and a distance from *B*, less by 11 chains than the length of the sewer: the expence of making it amounting to as many £s per chain as there were chains in the street leading to *A*. The sewer, however, being insufficient to carry off the water, an additional drain was made from a point in this street, distant 4 chains from the bridge *A*, which entered the river at the same point with the sewer, and was equally inclined to the river and sewer. Now it was found that a drain down the middle of each street, at the rate of £9 per chain, would have cost only £54 more than the expence of the sewer. Required the lengths of the streets, and the sewer; and the distance of its mouth from the bridge *B*.

7. On the institution of Saving Banks, an industrious labourer, with his wife and children, saved each of them a certain number of pence in a decreasing Arith. Prog. The sum saved monthly was less by 3*s*. 3*d*. than would have purchased one-sixth of as many bushels of wheat, as the seventh child

saved pence; the price of wheat being such, that the sum saved by the eldest and the fifth child, augmented by 10*s.*, would buy two bushels. But wheat rising 2*s.* per bushel, and work being scarce, the family find the sum saved would not buy as much wheat as their former savings, by two bushels; when it appears that at this rate the sum annually saved would be less by 5 guineas than by the former. Now the two youngest dying, it is found that if the remaining members of the family saved each one shilling less than the oldest child had done before the rise of wheat, their monthly account with the bank would not be affected by the deaths of the two youngest. Of how many did the family consist? What were the sums saved by each? And the price of wheat?

1817.

1. $\dfrac{5x-1}{2} - \dfrac{7x-2}{10} = 6\frac{3}{5} - \dfrac{x}{2}.$

2. $\left.\begin{array}{l} \sqrt{y} - \sqrt{a-x} = \sqrt{y-x} \\ \sqrt{y-x} + \sqrt{a-x} : \sqrt{a-x} :: 5 : 2 \end{array}\right\}$

3. $5.\dfrac{3x-1}{1+5\sqrt{x}} + \dfrac{2}{\sqrt{x}} = 3\sqrt{x}.$

4. $\left.\begin{array}{r} \dfrac{y}{2x} + \dfrac{2}{3} \cdot \dfrac{y - \sqrt{x-1}}{y^2 - 2\sqrt{x^2-1}} = \dfrac{\sqrt{x+1}}{x} \\ y^2 x - 1 = \dfrac{y^4}{4} \end{array}\right\}$

5. A farmer laid up a stock of corn, expecting to sell it in 6 months at 3*s.* per bushel more than he gave for it. But the price of corn falling one shilling per bushel, he found that by selling it at the expected time, he should lose the prime cost of five bushels. He therefore kept it till the end of the year, and selling it at two shillings per bushel under prime cost, found his loss to be ten shillings less than his expected gain. Required

the quantity of corn laid up, and price per bushel, allowing five per cent. simple interest.

6. A ship and crew of 175 men set sail with a store of water sufficient to last to the end of the voyage. But in thirty days the scurvy made its appearance, and carried off 3 men every day, and at the same time a storm arose, which protracted the voyage 3 weeks. They were, however, just enabled to arrive in port, without any diminution in each man's daily allowance of water. Required the time of the passage, and the no. of men alive when the vessel reached harbour.

7. The hold of a vessel partly full of water (which is uniformly increased by a leak) is furnished with two pumps worked by A and B, of whom A takes three strokes to two of B's, but four of B's throw out as much water as five of A's. Now B works for the time in which A alone would have emptied the hold, A then pumps out the remainder, and the hold is cleared in thirteen hours and twenty minutes. Had they worked together, the hold would have been emptied in three hours and forty-five minutes, and A would have pumped out 100 gallons more than he did. Required the quantity of water in the hold at first, and the horary influx at the leak.

1818.

1. $$\frac{4x-21}{7} + 7\tfrac{5}{6} + \frac{7x-28}{3} = x + 3\tfrac{3}{4} - \frac{9-7x}{8} + \frac{1}{12}.$$

2. $$\left.\begin{aligned} &\frac{3x-2y}{3} + 1 + \frac{11y-10}{8} = \frac{4x-3y+5}{7} + \frac{45-x}{5} \\ &45 - \frac{4x-2}{3} = \frac{55x+71y+1}{18} \end{aligned}\right\}$$

3. $$x^{\frac{7}{3}} + \frac{41\sqrt[3]{x}}{x} = \frac{97}{\sqrt[3]{x^2}} + x^{\frac{5}{6}}.$$

4. $$\left.\begin{aligned} &\frac{x^2y^2}{4} + 4 - 40y^2 = 140 - y^2\sqrt{x^2 - \frac{272}{y^2}} \\ &x^2 - \frac{2}{y}\left(\frac{3}{y} + 15x\right) = \frac{30}{y^2} + \frac{5x}{y} \end{aligned}\right\}$$

5. On January 1, 1799, a certain beggar received from *A* as many groats as *A* was years old, who repeated a similar donation every January for the seven following years, during the last of which *A* died, his alms to the poor man having in all amounted to £7 18*s*. 8*d*. Required in what year he was born, and his age at his death.

6. *A* entered into a canal speculation with fourteen others, and the profits of this concern amounted in all to £595 more than five times the price of an original share. Seven of his former partners in this affair joined him in a scheme for navigating the said canals with steam-boats, each venturing a sum of money less than his former gains by £173. But the steam-boats unexpectedly blowing up, *A* found he had lost £419 by them, for the company not only never recovered the money advanced, but had lost all they had gained by digging the canals, and £368 besides. What were the prices of shares in the two concerns originally?

7. *A*, *B*, and *C*, were three architects. *A* and *B* built four warehouses with flat roofs, each a large one, and each a small one, the linear width of the two large ones being the same, and also that of the two small ones. *A* built his as long and as high as they were wide, but *B* made the length and height of his large one equal to the width of his small one, and the length and height of his small one equal to the width of his large one, in such a manner that the difference between the solid content of those built by *A* and those built by *B* was 73,728 cubic feet. *C* also built a warehouse upon a square plot of ground which was equal to the difference between the ground plots occupied by those which *A* built, and found that it would have just stood on 2,688 square feet, if he had added eight

times as many square feet to the ground plot as there were linear feet in its width. How many feet wide were the several buildings erected by A, B and C?

1819.

1. $x - \frac{x-2}{3} = 5\frac{3}{4} + \frac{10+x}{5} + \frac{x}{4}$.

2. $$\left.\begin{aligned} 16x + 6y - 1 &= \frac{128x^2 - 18y^2 + 217}{8x - 3y + 2} \\ \frac{10x + 10y - 35}{2x + 2y + 3} &= 5 - \frac{54}{3x + 2y - 1} \end{aligned}\right\}$$

3. $mqx^2 - mnx + pqx - np = 0$.

4. $\frac{8}{3}\sqrt{\frac{4x}{5} + 17} - \frac{3}{4}\sqrt{6x + 16} - 2\sqrt{\frac{2x}{9} + 2\frac{2}{9}} = 0$.

5. A and B gained by trading £100. Half of A's stock was less than B's by £100, and A's gain was $\frac{3}{20}$ths of B's stock. What did each put into stock, and what are the respective shares of the gain?

6. Two labourers, A and B, whose rates of working were as 3 : 5, were employed to dig a ditch. A worked 12 hours and B ten hours a day; B being called away, A worked one day alone in order to complete the work; when they were paid, B received as many pence more than A as the no. of days they worked together. Now had B been called away a day sooner, A would have received 3*s.* 11*d.* more than B at the conclusion of the work. Required their respective daily wages, on supposition that the payment to each was in proportion to the work performed.

7. The 3 sides of a triangle AB, BC, CA are in Arith. Prog., and the difference of the squares of the first or shortest side AB and the common difference of the sides, is to the

square of the line bisecting the angle contained by AB, BC and terminated by CA as $25 : 18$; also the sum of the squares of the sides is 116. Required the sides of the triangle.

1820.

1. $$\frac{3x-4}{20} - \frac{70-8x}{12} - \frac{3-\frac{x}{4}}{5} = \frac{9-x}{2} - \frac{11-x}{15}.$$

2. $$\left.\begin{aligned} \frac{xy+86}{3x+2y} &= \frac{4x-y}{5} \\ \frac{xz+4\frac{1}{2}}{x+3z} &= \frac{x-z}{2} \\ \frac{yz+4\frac{1}{7}}{2y+z} &= \frac{y+3z}{7} \end{aligned}\right\}$$

3. $$\frac{3\sqrt{x}-x^{\frac{3}{2}}}{x+2} = \frac{1\frac{2}{3}+3\sqrt{x}-2x}{2\sqrt{x}-3}.$$

4. $$\left.\begin{aligned} (x-2)\,y - \sqrt{xy}\,(y^3-1) &= 2y^2 - x \\ \frac{xy}{4} &= \frac{\sqrt{xy}-12}{xy-18} \end{aligned}\right\}$$

5. A farmer at a fair found the price of an ox equal to that of 3 sheep, and that he could just dispose of £100, buying twice as many sheep as oxen. But waiting till the evening, when the price of an ox fell £1, and of a sheep 6*s*. 8*d*., he got for £100 three times as many sheep as oxen, and increased his whole stock by 10 more than he would have done in the former case. How many sheep and oxen did he buy, and what was the price of each?

6. There is a no. whose 3 digits are in Arith. Prog. If it be multiplied by the middle digit, and divided by the common

difference, the product is 548. The product of the extreme digits : square of the mean :: 15 : 16, and if 35 times the difference of the extreme digits be added to the no., its digits will be inverted. Required the no., its digits, and the radix of the scale in which it is expressed.

7. In going to town last winter by the Telegraph, I observed that we met the Union (which starts from town at the same time) an hour and a half after passing Royston, but in returning found that the Union had passed Royston just 40 minutes before the time of our meeting. The coachman told me, the roads were then very heavy, but that in summer when each coach travelled two miles an hour faster, he usually in going to town met the Union $\frac{8}{9}$ths of a mile nearer Cambridge, and left Royston two hours and a half before that coach arrived there. What were the respective distances of London, Cambridge and Royston from each other, and in what time did each coach perform its journey?

1821.

1. $$\frac{7x+5}{23} + \frac{9x-1}{10} - \frac{x-9}{5} + \frac{2x-3}{15} = 23\tfrac{1}{3}.$$

2. $$\left.\begin{array}{l} \frac{1}{2}\sqrt{x+4y} + \frac{1}{2}\sqrt{x+y} = \sqrt{x+2y} \\ \frac{3}{4}\sqrt{x^2-6y} + \sqrt{y^2+9x} : \sqrt{x^2-6y} :: 1\frac{3}{4} : 1 \end{array}\right\}$$

3. $$2\sqrt{3x+7} = 9 + \sqrt{2x-3}.$$

$$\left.\begin{array}{l} \dfrac{3a^2y^2 - \frac{4}{3}d^4}{3bx} = 3ay - \dfrac{8bx + 14d^2}{9} \\ \dfrac{2b^2x^2 + abxy - 3a^2y^2}{d^2} = 6d^2 - 6\frac{1}{2}bx - 13\frac{1}{2}ay \end{array}\right\}$$

5. A stage coach carries 6 inside, the fare outside is 13*s.* and $\frac{1}{3}$rd of the sum of the outside fares exceeds $\frac{1}{5}$th of those

inside by £1 1*s.* $8\frac{4}{5}$*d.* An opposition arising, the coachman loses 3 outside and 2 inside passengers, and also reduces the inside fare by 5*s.* and halves the outside; and then the whole loss is £7 0*s.* 6*d.* Find the number of outside places, and the fare inside.

6. A subscription is made for an Observatory, which is £900 less than the estimated expenses. The prices of the land building, and instruments are in an increasing Geom. Prog., the middle term being £1400 less than half of the sum subscribed. But if the money be put out at 5 per cent. simple interest, and the estimated sums be drawn out from the principal at the end of the 1st, 2nd and 3rd years respectively, there will remain £60 over. Required the sum subscribed, and the prices of the land, building, and instruments.

7. A steam boat sets out from London 3 miles behind a wherry, and having got to the same distance ahead, it overtakes a barge *floating* down the stream, and reaches Gravesend $1\frac{1}{2}$ hours afterwards. Having waited in order to land the passengers $\frac{1}{5}$th of the time of coming down, it starts to return, and meets the wherry in three quarters of an hour, the barge being then $5\frac{1}{4}$ miles ahead of the steam boat, and arrives at London in the same time that the wherry was in coming down from thence. Required the distance between London and Gravesend, and the rate of each vessel.

1822.

1. $$\frac{\frac{3x}{2}-4}{6}-\frac{4x-7}{9}+x=\frac{8-\frac{x+4}{2}}{3}+2.$$

2. $$\left.\begin{aligned} &\frac{x-6}{7y}+\frac{4x+7}{24}-\frac{x-\frac{y}{7}}{6}=\frac{19+y}{42}-\frac{\frac{11x}{3}+6}{56y} \\ &12x-15y+\frac{13}{4} : 10y-8x+\frac{86}{3} :: 93-9x : 6x-\frac{14}{5} \end{aligned}\right\}$$

3. $2x^{\frac{3}{2}}(x^3+a^3)^{\frac{1}{2}} = 2x^3(x+2a) + a^2(x-a).$

4. $$\left.\begin{array}{l} \dfrac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}\left(x^{\frac{2}{3}}-1\right) + \dfrac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\left(2x^{\frac{2}{3}}-1\right) = \dfrac{4y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\left(y^{\frac{1}{3}}+x^{\frac{1}{3}}\right) \\ \qquad\qquad\qquad\qquad + \dfrac{3y}{x^{\frac{1}{3}}} + 2 \\ \dfrac{x^{\frac{4}{3}}}{y^{\frac{4}{3}}} - \dfrac{2x^{\frac{2}{3}}}{y} - \dfrac{2x^{\frac{1}{3}}}{y^{\frac{1}{3}}} = \dfrac{133}{36}\cdot\dfrac{1}{y^{\frac{2}{3}}} - \dfrac{2}{x^{\frac{1}{3}}} - \dfrac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}} \end{array}\right\}$$

5. A tenant agreed to pay his landlord $\frac{2}{3}$rds of the profits of a farm after deducting the expense of cultivation, and upon this agreement found that his own share was $\frac{1}{6}$th of the whole produce. Afterwards the expense of cultivation having fallen in the ratio of 3 : 2, and the value of his produce in the ratio of 5 : 3, he found that adhering to his agreement, he must pay his landlord £400. Required the original value of the produce.

6. Three persons *A, B, C,* went into a gaming house; the sums which they severally had were in a decreasing Geom. Prog.; upon quitting it they found that the sums, which they then had, were in a decreasing Arith. Prog.; that what *B* had remaining : what he had lost :: the sum : the difference of what he and *C* had at first; and that *C* had neither won nor lost. If *C* had won what *A* lost, he would then have had £64 more than *A* had remaining; also the whole sum which they had remaining : what they had lost :: 6 : 7. Required the sums which they had at first.

7. A certain sum was to be laid out in the funds, the prices of which were such that the interest of money in the 5 per cents., 4 per cents., and 3 per cents. was in a decreasing Arith. Prog., whose common difference was $\frac{1}{4}$. The money was bought part into the 5 per cents. and part into the 3 per cents. At the end of 8 years all the funds having risen 4 per cent., the whole was sold out, when it was found that the amount of

interest and gain was the same as it would have been, if the whole had been originally bought into the 4 per cents.; and that by the whole transaction, £500 more was gained than would have been, if the whole sum had been put out to interest at 5 per cent. per annum. Also if the same sum had been bought into the 3 per cents. as was bought into the 5 per cents., and vice versa, the annual income would have been increased £109$\frac{3}{8}$. Required the whole sum laid out, the parts bought into the 5 per cents. and the 3 per cents., and the interest of money on the 5's, 4's, and 3's.

1823.

1. $$\frac{2x}{3} - \frac{1 - \frac{x}{2}}{4x} = \frac{x-1}{2} + \frac{x}{6} + \frac{7}{12}.$$

2. $$\left.\begin{aligned} x + 2y + 3z &= 17 \\ 2x + 3y + z &= 12 \\ 3x + y + 2z &= 13 \end{aligned}\right\}$$

3. $$a^2 b^2 x^{\frac{1}{n}} - 4(ab)^{\frac{3}{2}} x^{\frac{m+n}{2mn}} = (a-b)^2 x^{\frac{1}{m}}.$$

4. $$\left.\begin{aligned} x^4 + y^4 &= 1 + 2xy + 3x^2y^2 \\ x^3 + y^3 &= 2y^2x + 2y^2 + x + 1 \end{aligned}\right\}$$

5. A pedestrian finding that he could walk forwards 4 times as fast as he could backwards, undertook to walk a certain distance ($\frac{1}{4}$ of it backwards) in a given time. But the ground being bad, he found that his rate per hour backwards was $\frac{1}{5}$th of a mile less than he had supposed, and that to have won his wager, he must have walked forwards 2 miles an hour faster than he did. What is his rate per hour backwards?

6. *A* lent *B* a sum of money to be repaid with interest at the end of a year, and received as security Spanish 5 per cent. bonds to such an amount that their interest was equal to the

interest of the debt. At the year's end, *B* proved insolvent and Spanish bonds having fallen .40 per cent., *A* found that he had lost £400. Had they not fallen in value, he would have been enabled to repay himself and to return to *B* £250; and had he been at liberty to have sold them out at the half year's end, when they were at 50 (which was before the interest on them was payable) he would have lost only £300. Required the amount of the debt and its interest, and the price of Spanish bonds at the beginning and end of the year.

7. When a treadmill was erected in a gaol, the builder agreed to accept, instead of payment, the produce of (n) week's labour of the convicts *then* in the prison, and to supply them with food during that period. His estimate of the weekly expense of each man was (a) shillings; but finding this sufficient for the first week only of a man's labour, he increased it the second week by (ra), the third by (r^2a) and so on. Now at the beginning of each week after the first, (c) fresh convicts were sent to the mill, when he found that, had his contract included these, he would have gained as much as he had calculated on *at first*. Supposing the labour of each man to be equivalent to (pa) shillings, find the number of prisoners in the mill at first.

1824.

1. $$\frac{11x-13}{25}+\frac{19x+3}{7}-\frac{5x-25\frac{1}{3}}{4}=28\tfrac{1}{7}-\frac{17x+4}{21}.$$

2. $$\left.\begin{aligned}\frac{3}{x}-\frac{4}{5y}+\frac{1}{z}&=7\tfrac{3}{5}\\ \frac{1}{3x}+\frac{1}{2y}+\frac{2}{z}&=10\tfrac{1}{6}\\ \frac{4}{5x}-\frac{1}{2y}+\frac{4}{z}&=16\tfrac{1}{10}\end{aligned}\right\}$$

3. $$\frac{9\frac{3}{5} - \frac{3}{2}\sqrt{x} - x}{5\sqrt{x} - 8} + \frac{31}{50} = \frac{4}{5} \cdot \frac{\frac{74}{5}\sqrt{x} - x^{\frac{3}{2}}}{4x - 7}.$$

4. $$\left.\begin{aligned} \left(\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}}\right)^2 + \sqrt{y}\left(\sqrt{x} - \frac{\sqrt{y}}{2}\right) &= \frac{x}{\sqrt{y}}\left(\sqrt{x} - \frac{y}{\sqrt{2}}\right) \\ \frac{9\sqrt{x}}{\sqrt{y}} + \frac{3\sqrt{y}}{\sqrt{x}} &= (21\sqrt{2x} - 1)\frac{\sqrt{y}}{2\sqrt{x}} + \frac{1}{2\sqrt{xy}} \end{aligned}\right\}$$

5. A merchant wishing to buy a certain quantity of pimento, the price of which he calculates at the rate of 5 bags for £8, transmits to his foreign agent the requisite sum of money. Before the order arrives, pimento has risen in value, and the money is sufficient only to buy a quantity less by 18 bags than that which the merchant intended. It appears also, that as many bags as exceed $\frac{1}{3}$rd of the intended quantity by $5\frac{1}{4}$ will now cost £10 7*s.* more than they would have done had the price not varied. What is the quantity purchased?

6. From the towns *C* and *D*, two travellers *A* and *B* set out to meet each other: *A* beginning his journey 3 hours sooner than *B*. They meet at the distance of 20 miles from *D*, and *A* reaches *D* one hour before *B* arrives at *C*. Next day, *B* having begun to return at an early hour, meets *A*, who had then performed only $\frac{1}{7}$th of his journey back; and notwithstanding a subsequent delay of 3 hours arrives at *D* soon enough, were it necessary, to go 28 miles further before *A* reaches *C*. Required the distance between the towns and the rate at which each person travels.

7. A person in embarrassed circumstances called a meeting of his creditors, whose claims formed an increasing Arith. Prog. Upon examination, his effects were found sufficient to produce a dividend of as many shillings in the £ as there were £s in the common difference; and the amount received by the last

creditor would in that case, be equal to half the sum of the debts of the first and second. But the last creditor failing to establish the validity of his claim, the dividend to the rest was in consequence 2*s*. 8*d*. in the £ more than it would have been; and the sum thus lost by him was to that which the second now received as 45 : 28. Find the several debts and the value of the effects.

1825.

1. $$\frac{2x + 8\frac{1}{2}}{9} - \frac{13x - 2}{17x - 32} + \frac{x}{3} = \frac{7x}{12} - \frac{x + 16}{36}.$$

2. $$\left.\begin{array}{l} \frac{9}{8}\frac{\sqrt[3]{x + y}}{y} + \frac{9}{8}\frac{\sqrt[3]{x + y}}{x} = \frac{1}{7} \\ \frac{7}{4}\frac{\sqrt[3]{x - y}}{y} - \frac{7}{4}\frac{\sqrt[3]{x - y}}{x} - \frac{1}{9} \end{array}\right\}$$

3. $$\sqrt[2pq]{x^{p+q}} - \frac{1}{2}\cdot\frac{a^2 - b^2}{a^2 + b^2}\left(\sqrt[p]{x} + \sqrt[q]{x}\right) = 0.$$

4. $$\left.\begin{array}{l} 3x - x\sqrt{\frac{5x^2}{4} - 2y + 8} = 2 - y \\ \frac{\sqrt{x + y}}{2x} - \frac{3x}{4} = \frac{2x - 3}{\sqrt{x + y}} - \frac{3y}{2x} \end{array}\right\}$$

5. A farmer's rent was £50 a year, and his annual expenditure (including the assessed taxes, which amounted to $\frac{1}{6}$th of his expenses) was such that he was able to pay his landlord only £30. The year following, his rent was lowered 20 per cent.; the taxes also were reduced one half, and agricultural produce increased in value one third; in consequence, he was enabled to pay his rent and former debt and to lay by £5. What was his expenditure, and the value of his produce each year?

6. *A* sets out to ride from Newmarket to London at the same time that *B* and *C* leave Hockeril and London to ride to Newmarket. *A* meets *B* 4 hours before *C* overtakes *B*; but *A*,

on his return from London, meets C one hour before he meets B on their way from Newmarket. It was observed that A rode 10 miles an hour, and met B at the same place in going and returning. It is required to find the rates of travelling of B and C, and the distance from London to Newmarket; it being given that Hockeril is equally distant from each.

7. There are two vessels P and Q, containing quantities of fluid in the ratio of $4 : 21$, which consist of mixtures of wine and spirits in different proportions. A pumps a certain quantity out of P into Q; and then B pumps out into Q $\frac{3}{4}$ths of what was left; the strength of the mixture Q is then found to be $\frac{12}{13}$ths of its original strength. Now if when A stopped, B had pumped the same quantity as before out of Q into P, instead of from P into Q, the strength of the mixture P would have been exactly a mean proportional between the original strengths of P and Q, and B would have pumped out the same quantity of wine that he did before of spirits. Find the proportions of the wine and spirits in each of the vessels at first; and compare the quantities pumped out by A and B, the strength of the spirits being supposed 3 times that of wine.

1826.

1. $$\frac{25 - \frac{x}{3}}{x + 1} + \frac{16x + 4\frac{1}{5}}{3x + 2} = 5 + \frac{23}{x + 1}.$$

2. $$\left.\begin{array}{l} \dfrac{4x - 8y + 5}{2} = \dfrac{10x^2 - 12y^2 - 14xy + 2x}{5x + 3y + 3} + 2 \\ \sqrt{6 + x} : \sqrt{6 - y} :: 3 : 2 \end{array}\right\}$$

3. $$(a^{4b} + 1)\ (x^{\frac{1}{2}} - 1)^2 = 2(x + 1).$$

4. $$\left.\begin{array}{l} 2x + \sqrt{x^2 - y^2} = \dfrac{14}{y}\left(\sqrt{\dfrac{x + y}{2}} + \sqrt{\dfrac{x - y}{2}}\right) \\ \left(\dfrac{x + y}{2}\right)^{\frac{3}{2}} + \left(\dfrac{x - y}{2}\right)^{\frac{3}{2}} = 9. \end{array}\right\}$$

5. There was a run during the late panic on two bankers, A and B. B stopped payment at the end of 3 days, in consequence of which the alarm increased, and the daily demand for cash on A being trebled, A failed at the end of two more days. But if A and B had joined their capitals, they might both have stood the run as it was at first, for seven days, at the end of which time B would have been indebted to A £4000. What was the daily demand for cash on A's bank at the beginning of the run?

6. The gas contractors engage to light a *shop* with 5 large and 3 small burners, but having by them only one large burner, they supply the deficiency with 5 small ones. The shopkeeper, not finding this light sufficient, procures 2 more small burners, and at the same time agrees for the lights to burn double the usual time on Saturday nights, for which additional gas he was required to pay £1 11*s*. How much did he pay a year altogether?

7. A man, who is not aware that his watch gains uniformly, engages to ride from Cambridge to London in 9 hours, and sets his watch by St Mary's at the time of starting. Upon looking at his watch after having gone half way, he supposes it necessary to increase his pace in the ratio of 4 : 3, in consequence of which he arrives in London a quarter of an hour within the time agreed on. But if the watch had lost at the same rate, and he had looked at it at the 14th milestone, and then regulated his pace accordingly, he would have been in London too late by 7 minutes. Find at what rate he set out, and the distance from Cambridge to London by the road he travelled.

1827.

1. $$\frac{x - 1\frac{25}{26}}{2} - \frac{2 - 6x}{13} = x - \frac{5x - \frac{10 - 3x}{4}}{39}$$

2. $\sqrt{y} - \sqrt{y-x} = \sqrt{20-x}\ \sqrt{y-x} : \sqrt{20-x} :: 3 : 2.$

3. $8\sqrt{3x} + \dfrac{243 + 324\sqrt{3x}}{16x - 3} = 16x + 3.$

4. $\left.\begin{array}{l} x + \sqrt{3y^2 - 11 + 2x} = 7 + 2y - y^2 \\ \sqrt{3y - x + 7} = \dfrac{x+y}{x-y} \end{array}\right\}$

5. From each of two bags containing a certain number of balls respectively, a person draws out a handful, and finds that the number remaining in the greater is exactly the cube of that remaining in the lesser, and exactly the square of one handful. He then draws out of the greater until he finds that the number remaining in it is exactly the square of that remaining in the lesser, and also that if he now empties the greater into the lesser, its original number will be increased by two-thirds. Required the number of balls in each bag.

6. A and B are two towns situated on the bank of a river which runs at the rate of 4 miles per hour; a waterman rows from A to B and back again, and finds that he is 39 minutes longer upon the water than he would have been had there been no stream. The next day he repeats his voyage with another waterman, with whose assistance he can row half as fast again; and they find that they are only 8 minutes longer performing their voyage, than they would have been had there been no stream. Required the rate at which the waterman would row by himself.

7. Two master bricklayers undertake to lay the foundation of a new court, each taking a certain part and begin at the same time. If they had continued to work together until the whole was finished, it would have required only $\frac{4}{5}$ths of the time it actually took; and in this case B would do enough to occupy A three months, and A enough to occupy B twelve months, which is 36 yards more than A contracted to do. How many yards did the foundation contain?

1828.

1. $$\frac{6x - 7\frac{1}{3}}{13 - 2x} + 2x + \frac{1 + 16x}{24} = 4\tfrac{5}{12} - \frac{12\frac{5}{8} - 8x}{3}.$$

2. $$\left.\begin{aligned} x(bc - xy) &= y(xy - ac) \\ xy(ay + bx - xy) &= abc(x + y - c) \end{aligned}\right\}$$

3. $$16(x^2 + 2)^{\frac{3}{2}} + \frac{3}{\sqrt{x^2 + 2}} = 32x^2 + 48.$$

4. $$\left.\begin{aligned} &30\sqrt{\frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{x^{\frac{4}{3}}y^{\frac{4}{3}}}} + 40\sqrt{\frac{x^{\frac{2}{3}}y^{\frac{2}{3}}}{x^{\frac{2}{3}} + y^{\frac{2}{3}}}} = 241 \\ &\left\{1 + \left(\frac{y}{x}\right)^{\frac{2}{3}}\right\}\left\{3x^{\frac{4}{3}}y^{\frac{2}{3}} + \frac{91}{216}\sqrt{x^2 + x^{\frac{4}{3}}y^{\frac{2}{3}}}\right\} \\ &\qquad = \left(\frac{5}{6}\right)^3 - x^2 - y^2 \end{aligned}\right\}$$

5. Two clocks are striking the hour together, and are heard to strike 19 times. There is a difference of two seconds in their time, and one strikes every 3, the other every 4 seconds. What is the hour they strike? It being observed that when the clocks strike in the same second, the sounds cannot be distinguished, so as to determine whether one or both strike in that second; and that this is the case with the last stroke of the faster clock.

6. A revenue cutter observes a smuggler q leagues directly to windward; and gives chase, sailing at $5\frac{1}{3}$ points from the wind, and making tacks of $4p$ miles. The smuggler lies off on the other tack at $2\frac{2}{3}$ points, making tacks of $\frac{1}{3}p\sqrt{3}$ miles, its rate of sailing being to the cutter's as $1 : 4\sqrt{3}$. They sail half the above distances before the first tack. In what tack will the smuggler, while lying in the eye of the wind, first be within range of the cutter's guns, which carry r miles?

7. In the first irruption of the Thames into the Tunnel,

the water rose in the vertical shaft 8 times as fast as in the horizontal levels in the second irruption. It was observed also that if the levels at the second influx had been 110 feet longer, the velocities of the water ascending in them, in the first and second irruptions and when thus increased, would have formed an Arith. Prog., the common difference of which is $\frac{1}{9}$th of the difference of the velocities with which the water rose in the shaft in the two irruptions; and had the levels been of the same length at the first as at the second influx, the whole time of filling would have been half as long again. The tunnel consists of two equal levels, terminated by a vertical shaft of twice their breadth; the sections of the shaft and levels are supposed to be squares, and the height of the shaft above the upper surface of the levels to equal twice its breadth. The time of filling in the first irruption being 10 minutes less than that in the second; find the time in the second, and the dimensions of the tunnel.

1829.

1. $$\frac{7x+6}{28} - \frac{2x+4\frac{2}{7}}{23x-6} + \frac{x}{4} = \frac{11x}{21} - \frac{x-3}{42}.$$

2. $$\left.\begin{array}{c} \sqrt{x-y} + \frac{1}{2}\sqrt{x+y} = \dfrac{x-1}{\sqrt{x-y}} \\ x^2 + y^2 : xy :: 34 : 15 \end{array}\right\}$$

3. $$\frac{x^{(m-n)^2} + x^{-4mn}}{x^{(m-n)^2} - x^{-4mn}} = a^{\frac{r}{s}}.$$

4. $$\left.\begin{array}{c} 5 - 2\sqrt{y+2} = \dfrac{9x^2}{64} - (\sqrt{x} - 3\sqrt{y})^2 \\ \dfrac{7}{y} - 10\sqrt{\dfrac{x}{y}} = x - 16. \end{array}\right\}$$

5. 600 persons voted on a disputed question, which was lost by a certain no. The same no. of persons having voted again on the same question, it was, from some change of

circumstances, carried by twice as many as it was before lost by; and the new majority was to the former one as 8 : 7. How many changed their minds?

6. There are 3 towns A, B, C, the straight lines joining which form a right angled triangle, B being situated at the right angle, and the distance from A to B being the least of the three; a pedestrian making a circuit of them, at a uniform rate, finds that the time of his going from A to B together with the time of going from B to C, exceeds the time from C to A by 2 hours and 40 minutes. A coach which left A 4 hours after the pedestrian to make the same circuit, overtakes him at the end of the 8th mile from B to C, the rate of the coach being three times that of the pedestrian; and after reaching A, and waiting there 6 hours and 40 minutes, it starts again to make the same circuit, and arrives again at A, exactly at the same time with the pedestrian who had rested 4 hours at C. Find the distances of the towns from each other, and the rates of travelling of the pedestrian and coach.

7. A, B, C, D, are 4 rough diamonds, the value of C in £s is 52 less than the weight of A in carats, and the value of C and D in £s is equal to the weight of B in carats; after being cut, each is found to have lost half its weight. The dust from A and B is worth £85; the value of A is to the value of C, D and the dust from A as the value of B is to the value of dust from B. A diamond weighing one carat, when rough, is worth £3 when cut, and £2 when uncut; the value is proportional to the square of the weight and a carat of the dust is worth £1. Find the value of D when cut.

1830.

1. $$\frac{6-5x}{15} - \frac{7-2x^2}{14(x-1)} = \frac{1+3x}{21} - \frac{2x - \frac{11}{5}}{6} + \frac{1}{105}.$$

2. $$\left.\begin{aligned} &3x + \frac{2}{3}\sqrt{xy^2 + 9x^2y} = \left(x - \frac{1}{3}\right)y \\ &6x + y : y :: x + 5 : 3 \end{aligned}\right\}$$

3. $$\sqrt{x} - \frac{8}{x} = \frac{7}{\sqrt{x} - 2}.$$

4. $$\left.\begin{aligned} &x^2y - 4 = 4x^{\frac{1}{2}}y - \frac{y^3}{4}. \\ &x^{\frac{3}{2}} - 3 = x^{\frac{1}{2}}y^{\frac{1}{2}}\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right) \end{aligned}\right\}$$

5. Two persons A and B, start at the same time for a race, which lasted 6 minutes. Now after galloping 4 minutes at the same uniform pace at which each started, the distance between them is $\frac{1}{440}$th part of the whole length of the course. They continue to run for 1 minute more at the same speed as at first; and then B, who is last, quickens the speed of his horse 20 yards a minute, and comes in exactly 2 yards before A, whose horse has run at the same uniform pace throughout. What was the length of the course?

6. The revenue of a state was increased, to provide for a war, in the ratio of $2\frac{1}{4} : 1$; and after deducting the expence of collecting and the interest of the National Debt, the available income was augmented in the ratio of $3\frac{12}{23} : 1$. Now, it was found upon calculation, that had circumstances, on the contrary, permitted the revenue to be reduced in the ratio of $1\frac{7}{9} : 1$, the sum remaining, after the specified reductions, would have been diminished in the ratio of $7\frac{2}{3} : 1$; and would in fact have ouly amounted to 4 millions. Required the amount of the revenue and the interest of the debt; on the supposition, that the expence of collecting varies as the square root of the amount collected.

7. Three boats A, B, C, start in a race at the same instant; B being 20 yards behind A, and C the same distance behind B. A and C set off at a uniform rate, C advancing one yard less than A at every stroke. But B took 7 strokes to 6 of A's or

C's, and increased its speed besides by 3 inches every stroke; so that by the time A had taken 42 strokes, B, though it had lost 16 yards by steering, was only one yard behind A. At this time B was observed to fall back, and its velocity decreased twice as fast as it had increased before: whilst C quickening its strokes at the same instant in the ratio of 6 : 7 and gaining each stroke as much velocity as B lost, at the end of 28 strokes overtook B, which had lost 11 yards more by steering. Compare the velocities with which they started.

1831.

1. $$\left.\begin{aligned} x - \frac{2y - x}{23 - x} &= 20 - \frac{59 - 2x}{2} \\ y + \frac{y - 3}{x - 18} &= 30 - \frac{73 - 3y}{3} \end{aligned}\right\}$$

2. $$\left.\begin{aligned} &\left(a^2 - a\sqrt{b^2 + bx} + \frac{bx}{2} - \frac{b^2}{4} + \frac{b}{2}\sqrt{x^2 - bx + b^2}\right) \times \\ &\left(a^2 + a\sqrt{b^2 + bx} + \frac{bx}{2} - \frac{b^2}{4} - \frac{b}{2}\sqrt{x^2 - bx + b^2}\right) \\ &\qquad = a^4 - \frac{3b^4}{16} \end{aligned}\right\}$$

3. $2x\sqrt{1 - x^4} = a(1 + x^4)$.

4. $$\left.\begin{aligned} (2 + 4xy - 3x^2)^2 &= 2 - 4x^2y^2 + 3x^4 \\ (x^2 - 1)^2 &= (2y^2 + x^2 + 1)(2y^2 - x^2 - 1) \end{aligned}\right\}$$

5. The owner of a balloon calculated that if he filled the enclosure, which he has hired for the day at £5, with spectators at 2*s*. a head, and two persons ascended with him, he should gain $\frac{7}{5}$ths of his outlay. The gas, and the weather proving bad, he pays but half the price of inflating, and ascends alone from the enclosure a fourth part full and loses $\frac{1}{3}$rd of his outlay. He ascends again with a full balloon, the enclosure $\frac{3}{4}$ths filled and a

companion with him. By the whole speculation he gained £10. What did it cost him to fill his balloon?

6. *A* and *B* row between two places, *B* in a time during which the minute hand of his watch moves over a certain space; but when the minute hand of *A*'s watch has described an equal space, he is obliged to relax his speed and for the rest of the distance moves only $\frac{2}{3}$rds as fast as before. When the stream, which flows at a given uniform rate, is in their favour, the first part of the distance takes *A* six times as long as the last; but when the stream is against them, the two parts are performed by him in equal times; they are also performed in equal times with an adverse stream even if *A*, instead of flagging, increase his speed in the ratio of 7 : 5, provided *A* and *B* exchange watches at starting. Supposing their watches to gain uniformly, compare the velocities of *A* and *B*.

7. A regiment, in which there are between 10 and 100 sergeants, and half as many officers, in clearing the streets during a revolution, loses 2 officers, and after storming a barricade in which 3 more fall and one accidentally joins, is obliged to retreat and loses other 3; whilst engaged in clearing the streets, the liability of an officer to fall, is half that of a sergeant's or private's, but at the barricade as 4 : 3, and during the retreat as 3 : 4; also on leaving their barracks, the no. whose left-hand digits are the sergeants, and right the officers, is 20 more than 10 times the no. of privates, but on coming back it is only 12 more, the no. of sergeants being still greater than 10. Required the state of the regiment at first.

1832.

1. $$\frac{2}{x} - \frac{1}{4} \cdot \frac{15 - 2x}{1 + 2x} = \frac{x^2 + 1}{4x^2 - 1}.$$

2. $$\left.\begin{aligned} &\sqrt{x} - \sqrt{y} = \sqrt{x}\,(\sqrt{x} + \sqrt{y}) \\ &(x + y)^2 = 2\,(x - y)^2 \end{aligned}\right\}$$

3. $\sqrt{x} \,.\, \sqrt{x^3 + a^3} = x^2 - a^2 + 3ax.$

4. $$\left.\begin{aligned} \sqrt[3]{\frac{27y^{\frac{3}{2}} - 1}{x^3 + 3y^2 - 2xy^{\frac{3}{2}}}} &= 3\sqrt{\frac{x}{y}} \\ 3x^2 + 42xy + 16y^2 &= 4\sqrt{xy}\,(5x + 11y) \end{aligned}\right\}$$

5. A farm was rated at 3*s*. an acre, and the tenant on receiving back at his rent day 10 per cent. of his rent, found that the sum returned amounted to £6 more than the whole rate. The next year the rates were doubled, and he received back 15 per cent. of his rent; but he now found that the sum returned only just paid for the whole rate. What was the rent of the farm and of how many acres did it consist?

6. A person sets off to walk from Cambridge to London at the rate of $3\frac{1}{2}$ miles an hour. In $2\frac{1}{2}$ hours he is overtaken by the Times, and at ten minutes before 10 o'clock by the Fly; after resting $2\frac{1}{2}$ hours on the road, he starts again and meets the Times on its return from London, and half a mile further the Fly, at 20 minutes past 5. Supposing the Times and Fly to have started from Cambridge at 6, and half-past seven o'clock respectively, and from London at 3; determine the distance from Cambridge to London, and the rates at which the coaches travelled.

7. At a contest for returning two members to parliament, one third of the electors gave plumpers for *C*, and those given for *A* and *B* were $\frac{4}{21}$ths of the whole no. of votes given. Of those electors who gave single votes to *C*, twice as many voted for *B* as for *A*. A scrutiny being demanded, it appears that those who had voted for *A* and *C* and half of those who had voted for *A* and *B*, had no legal right to vote; in consequence of which *C* is now placed at the head of the poll, with a majority of 20 over *A*. It is observed too, that *A* now has as many single votes as plumpers, and that his majority over *B* is diminished by one-half. Required the state of the poll at the close of the election.

1833.

1. $$\frac{3x-2}{4}+\frac{x}{2}-11\tfrac{5}{6}=\frac{x-\dfrac{4x-9}{3}}{6}-5.$$

2. $$\left.\begin{aligned}3y+11&=\frac{4x^2-y\,(x+3y)}{x-y+4}+31-4x\\(x+7)\,(y-2)+3&=2xy-(y-1)\,(x+1)\end{aligned}\right\}$$

3. $$x-2\sqrt{x+2}=1+\sqrt[4]{x^3-3x+2}\,.$$

4. $$\left.\begin{aligned}(1-x^2)^2\,(1+y^2)-(1+x^2)^2\,(1-y^2)&=4x^2\sqrt{1+y^4}\\4xy&=\sqrt{2}\,(1-x^2)\,(1-y^2)\end{aligned}\right\}$$

5. A and B engaged to reap equal quantities of wheat, and A began half an hour before B. They stopped at 12 o'clock, and rested an hour, observing that just half the whole work was done. B's part was finished at 7 o'clock, and A's at a quarter before 10. Supposing them to have laboured uniformly, determine the time at which they commenced.

6. Two boys set off in opposite directions from the right angle C of a triangular field, and ran along the sides without varying their velocities, which were in the ratio of 13 : 11. They met in the middle of the opposite side, and afterwards 30 yards from the point whence they started. Required the lengths of the sides of the field.

7. A stable keeper bought two horses for £50, and at the end of a year sold one of them for double and the other for half what he gave for it. The former, being well-fed and lightly worked, produced for its hire only the half of what it cost him, and consumed in keep as much per cent. on its price, as the hire of the other produced on its price, the latter being kept for $\frac{5}{8}$ths of as many guineas as it sold for pounds. The keep of the two amounted to £33, and the whole sum that he made by the horses was nine times his profit on the sale. What did each horse cost?

1834.

1. $$\frac{1}{3} - \frac{7x-1}{6\frac{1}{2} - 3x} = \frac{8}{3} \cdot \frac{x - \frac{1}{2}}{x-2}.$$

2. $$\left.\begin{aligned} x^m a^n + y^n b^m &= 2(ax)^{\frac{m}{2}}(by)^{\frac{n}{2}}. \\ xy &= ab \end{aligned}\right\}$$

3. $$x^2 - 2x + 4 = 2\sqrt{x^3 - 1}.$$

4. $$\left.\begin{aligned} a^2(x^4 + y^4) &= y^6 - 2a^2xy\sqrt{x^4 - y^4} \\ a^2(x^4 - y^4) &= x^2y^2(2a^2 - x^2) \end{aligned}\right\}$$

5. A and B start to run a race to a certain post and back again. A returning, meets B 90 yards from the post and arrives at the starting place 3 minutes before him. If he had returned immediately to meet B, he would have met him at $\frac{1}{6}$th of the distance between the post and the starting place. Find the length of the course and the duration of the race.

6. The upper spokes R and r of the hind and fore wheels of a carriage are vertical at starting. After r has made one revolution, its direction is at right angles to the spoke next before R; and when R has made $\frac{9}{8}$ths of a revolution, r, ascending through its second revolution, has the same inclination to the horizon as the spoke next before it. Given that the diameter of the forewheel : no. of spokes in it :: the difference of the heights of the axles : 2, compare the magnitudes of the wheels, and find the no. of spokes in each.

7. A landlord agrees with his steward to allow him a certain per centage on the rents he collected, on condition that he returns half the same per centage on the rents not paid. The first year, the steward's income amounts to six per cent. on the whole rental, but in the following he finds it necessary, in order to make up the deficiency from his last year's in-

come, to make a return of rents received £270 under their actual value. In the third year, though the whole rents are reduced $7\frac{1}{2}$ per cent., the amount of rents not paid is the same as in the second year; the steward's income is only $\frac{3}{4}$ths of his first year's income and to make up the deficiency, he doubles the amount of his last year's fraud. Required the rental of the estate.

1835.

1. $\frac{1}{2}\left(\frac{2x}{3} + 4\right) - \frac{7\frac{1}{2} - x}{3} = \frac{x}{2}\left(\frac{6}{x} - 1\right).$

2. $$\left.\begin{aligned} \frac{x}{3} + \frac{y}{4} + \frac{z}{5} &= 5\frac{2}{3} \\ \frac{y}{3} + \frac{x}{4} + \frac{z}{10} &= 5\frac{1}{2} \\ \frac{z}{3} + \frac{x}{5} + \frac{y}{6} &= 5\frac{11}{15} \end{aligned}\right\}$$

3. $\frac{1 + x^5}{(1 + x)^5} = a.$

4. $$\left.\begin{aligned} y + 3\sqrt[3]{y}\left(\sqrt[3]{a + bx} - \sqrt[3]{y}\right)\sqrt[3]{a + bx} &= 2a \\ \frac{y - \sqrt[3]{y}\cdot\sqrt[3]{a^2 - b^2x^2}}{\sqrt[3]{y} - \sqrt[3]{a + bx}} &= 2a\sqrt[n]{a - bx} \end{aligned}\right\}$$

5. Four cannons A, B, C, D, are fired at the same instant. The reports of B and D reached A, a half and a third of a second respectively before that of C; and the reports of A and C reached B a sixth and a quarter of a second before that of D. DC is $264\frac{18}{23}$ yards and the angle ADC is equal to that which BC makes with AB produced. Supposing sound to travel 360 yards in a second, determine the distances of the cannons from each other.

6. Three men A, B, C, of given unequal powers engage in a

partnership to reap a field of wheat. After (p) days a dispute arising C withdraws, and receives (q) shillings for his work, being his just portion after the deduction which the farmer has determined to make per acre from the stipulated price, on account of the delay thus occasioned. A and B finish the work (at the original price), when A receives (m) shillings more than he would have done if C had remained, and the farmer gains in money as much as he had at first agreed to pay per acre. Find the no. of acres, the deduction made, and the sums received by A and B.

7. P, Q, R, represent 3 candidates at an election. Q polled as many plumpers wanting one, as the split votes betwixt P and R exceeded those betwixt himself and R; and the no. of split votes betwixt Q and R was one more than twice the no. betwixt Q and P. If P had not voted for himself and R, but for R only, and if 5 others who split betwixt P and Q had voted for Q only, Q would have just beaten P, and would have been 48 below R. The no. of voters was 1341, of which 565 gave plumpers. Required the no. of plumpers for each candidate, and the final state of the poll.

1836.

1. $$\frac{7x - 13\frac{1}{2}}{11} - \frac{2}{3} \cdot \frac{x - 15}{7} = \frac{15}{14}(x - 1).$$

2. $$\left.\begin{aligned} \frac{2x}{x+y} - \frac{y^2}{x^2 - y^2} &= \frac{13x + 16y}{8(x + 2y)} \\ \sqrt{x} + \sqrt{y} &= \sqrt{x + y + \sqrt{3}} \end{aligned}\right\}$$

3. $$\sqrt{2(x + 2)} - 2\sqrt{2 - x} = \frac{12x - 8}{\sqrt{9x^2 + 16}}.$$

4. $$\left.\begin{aligned} &ax^2 + by^2 = a(x + b) \\ &x^4 + x^2(y^2 - 2x) - x(y^2 - 1) + y^4 = b \end{aligned}\right\}$$

5. In a race between two boats a spectator walking at the rate of 5 miles an hour, is $\frac{1}{8}$th of a mile a-head of the first boat at starting; and when it passes him, he observes that the interval between the boats, which at first was 30 yards is reduced to 20. At the distance of $1\frac{1}{4}$ miles from the place where it started the first boat is overtaken by the second; how long did the race last?

6. When wax candles are half-a-crown a pound, a composition is invented of such a nature, that a candle made of it will burn $\frac{2}{3}$rds of the time in which a wax candle of the same thickness and $\frac{1}{4}$th as heavy again will continue burning. Supposing the two candles give an equally bright light, what must be charged per lb. for the composition, that it may be as *cheap* as wax?

7. Into a cubical cistern, eight feet deep, and having an unknown leak, water is poured from two pumps, worked by two men, A and B. They pump together till the vessel is half filled, when B falls asleep; A continues pumping till it is three-fourths filled, and then goes away. B afterwards waking finds the cistern still half full, and after pumping till it is again three-fourths filled, departs also, and meeting with A charges him with leaving his work unfinished. They return together and find the water $1\frac{1}{2}$ inch lower than when B left. The leak is now discovered and stopped, and by their joint efforts the vessel is filled in half the time in which they had worked together at first. They remark also that $10\frac{1}{3}$ hours had elapsed since they first began pumping, and that B had worked alone twice as long as A had. Supposing that a cubic foot contains $15\frac{5}{8}$ gallons, find the quantity of water thrown in by each pump.

1837.

1. $$\frac{4x-17}{9} - \frac{3\frac{2}{3} - 22x}{33} = x - \frac{6}{x}\left(1 - \frac{x^2}{54}\right)$$

2. $$\left.\begin{aligned} x\,(y+z)^2 &= 1 + a^3 \\ x + y &= \frac{3}{2} + z \\ yz &= \frac{3}{16} \end{aligned}\right\}$$

3. $$4\left\{(x^2 - 16)^{\frac{3}{4}} + 8\right\} = x^2 + 16\,(x^2 - 16)^{\frac{1}{4}}.$$

4. $$\left.\begin{aligned} (x^6 + 1)\,y &= (y^2 + 1)\,x^3 \\ (y^6 + 1)\,x &= 9\,(x^2 + 1)\,y^3 \end{aligned}\right\}$$

5. A person at his death leaves a certain sum in money and debts to a certain amount. The executors invest the money in the funds, then at 96. Of the debts $\frac{1}{6}$th is not recovered; and when the stock is sold out at 92, it is found that the heir receives £140 less than he would have done if the debts had been completely recovered. His loss is also $\frac{1}{8}$th of the sum he receives. Find the amount of the debts and money.

6. A ferry boat, with a certain no. of passengers, was about to cross a river, when it was upset by a party leaping in, who increased the no. of persons in the boat in the ratio of 4 : 5. The no. who got out without assistance, including the ferryman, was $\frac{5}{6}$ths of the increased no. of passengers, and the no. taken out was $\frac{1}{3}$rd the no. of minutes the last man was in the water. The nos. extricated in both ways in each of the first three minutes successively form a series of fractions whose numerators increase in Arithmetical and denominators in Geometrical progression, the common ratio and difference being the same as the no. of persons in the water at the end of the 3 minutes and the first term being the no. of minutes remaining till the last man was out. Had, however, the no. of intruders been less by 4 and still increased the no. of persons in the boat in the same proportion as before, the increased no. of passengers would have been twice the common ratio of the denominators. How long was the last man in the water?

7. Towards the close of a cricket-match, the second party were a certain no. of notches behind their opponents, and had still 3 men, *A*, *B*, *C*, remaining. *A* and *B* are in, and after $\frac{5}{8}$ths of the no. have been gained, *A* is struck out, and *C* takes his place. Now *B* scores as many notches in *C*'s innings as there were bye balls in *A*'s; and as many in *A*'s, as were gained altogether in *C*'s. If, also, the byes in *A*'s innings be added to *B*'s notches in it, and the byes in *C*'s innings to *C*'s notches, these quantities will be inversely proportional to the corresponding nos. of byes. *C* gets one more notch than *B* in their common innings and the party loses by 3; though, if *B*'s scoring be reversed, *i. e.* if he be supposed to get as many notches in *A*'s innings as the no. of byes in *C*'s, and as many in *C*'s as the whole no. now gained in *A*'s, the three would have scored between them, without reckoning the bye balls, just as many as their whole former no. How many notches did *A* score?

1838.

1. $\frac{3x}{2} + \frac{81x^2 - 9}{(3x-1)(x+3)} = 3x - \frac{3}{2} \cdot \frac{2x^2 - 1}{x+3} - \frac{57 - 3x}{2}.$

2. $$\left.\begin{aligned} xy + z &= 5 \\ xyz &= 4 \\ 2(x^2 - y) &= (y^2 - x)^2 \end{aligned}\right\}$$

3. $(x+3)^2 - 2(x^2+3) = 2x(x+1)^2.$

4. $$\left.\begin{aligned} (x+y)^2 &= x^4 + x^2y^2 + y^4 \\ x^4 + 4y^4 &= 4xy(2y^2 - x^2) \end{aligned}\right\}$$

5. In a tithe commutation, the rent-charge was fixed at 3*s*. an acre, and the tithe owner found the first year that his rates wanted £6 of being 10 per cent. on his receipts. The next year the rates were doubled and amounted to 15 per cent. on his receipts. What was the no. of acres in the parish?

6. An omnibus starts with a certain no. of passengers and takes up four more on the road whose fare is the same as that paid by the others. On deducting $\frac{1}{12}$th of the whole fare for expences, there remains a gain of 4*s*. 7*d*. But if those who were taken in last had each paid half as many pence as there were passengers altogether, the money received would have exceeded double its difference from the sum actually paid by 2*s*. 8*d*. With how many did the omnibus start, and what was the fare of each?

7. *A* and *B* having a single horse travel between two mile-stones, distant an even number of miles, in $2\frac{62}{63}$ hours, riding alternately mile and mile, and each leaving the horse tied to a mile-stone until the other comes up. The horse's rate is twice that of *B*; *B* rides first and they come together to the 7th mile-stone. Finding it necessary to increase their speed each man after this walks half a mile per hour faster than before, and the horse's rate is now twice that of *A*, *B* again riding first. Find the rates of travelling and the distance between the extreme mile-stones.

1839.

1. $x^2 - 7 = \sqrt{x^2 - 42x + 89}.$

2. $$\left.\begin{aligned} y\sqrt{(a-x)(x-b)} + x\sqrt{(a-y)(b-y)} \\ = 2\left\{b\sqrt{(a-x)(a-y)} + a\sqrt{(x-b)(b-y)}\right\} \\ xy = 4ab \end{aligned}\right\}$$

3. $\dfrac{x}{2} + \dfrac{63}{\sqrt[4]{x}} = \dfrac{220\frac{1}{2}}{\sqrt{x}} + 49\sqrt{x} - 1196.$

4. $$\left.\begin{aligned} 8(x + y + xy) = (x^2 + y^2)^2 \\ \sqrt{x+y} + \sqrt{4-(x+y)} = \sqrt{3} + 1 \end{aligned}\right\}$$

5. $$\left.\begin{aligned} y^4 = x^2(ay - bx) \\ x^2 = ax - by \end{aligned}\right\}$$

6. A steam vessel leaves Oban for Staffa with a supply of whiskey (b) above proof, (which is assumed to mean that $(a+b)$ gallons of spirit are mixed with (c) gallons of water) sufficient for two days' consumption provided it receive no addition to its crew. On arriving at Tobermory (m) of its passengers remain behind, but by reason of contrary winds its progress to Staffa the following morning is retarded, so that on its return to Oban it is with difficulty enabled to reach Iona by midnight. It here receives (n) additional passengers, and also (p) gallons of whiskey (d) above proof. On an average each passenger dilutes his whiskey with water till it is (e) below proof, and consumes (q) pints of the mixture daily. The vessel arrives at Oban on the evening of the third day after its departure, by which time the supplies of whiskey are both exhausted. Required the no. of passengers on board when it left Oban, and the no. of gallons of whiskey in the first supply.

7. Fine gold chains are manufactured at Venice, and are sold at so much per braccio, a measure containing about two feet English. When there are 90 links in an inch, the value of the workmanship of a braccio is equal to the whole value of a braccio when there are but 30 links in an inch; and the whole value of the braccio in the former case is equal to three times the difference between the cost of the material and workmanship of a braccio in the latter, together with $4\frac{4}{9}$ francs. Supposing that the workmanship in each braccio varies as the no. of links in an inch, and the weight of metal inversely as the square of that no., find the values of the material and workmanship in a braccio of each of the chains.

8. There are 7 rows of quantities, the first being horizontal and consisting of the squares of a series of nos. in Arith. Prog.; the 2^{nd} row is formed so that the vertical difference between the n^{th} terms of the 1^{st} and 2^{nd} rows $= \frac{1}{2}$ the horizontal difference between the n^{th} and $(n+1)^{th}$ terms of the 1^{st} row; and any other even row as the $(2r)^{th}$ is formed so that the vertical

difference between any one of its terms as the n^{th} and the corresponding term of the ($2r - 1^{\text{th}}$ row $= \dfrac{7-r}{16-4r} \times$ the difference between the n^{th} and $(n+1)^{\text{th}}$ terms of the $(2r-2)^{\text{th}}$ row: but the $(2r+1)^{\text{th}}$ row is formed so that the vertical difference between the n^{th} terms of $(2r+1)^{\text{th}}$ and $(2r)^{\text{th}}$ rows $= \dfrac{r}{16-4r} \times$ the difference between the n^{th} and $(n+1)^{\text{th}}$ terms of the $(2r-1)^{\text{th}}$ row. The first terms of the fifth and sixth rows are $\dfrac{151}{24}$ and $\dfrac{281}{24}$. Find the first and seventh rows.

1840.

1. $$\frac{x}{2} - \frac{\dfrac{2x-3}{3} - \dfrac{3x-1}{4}}{\dfrac{x-1}{2}} = \frac{3}{2} \cdot \frac{x^2+2}{3x-2}.$$

2. $$\left.\begin{aligned} &\frac{\sqrt{y^2+1}+1}{y} = \frac{\sqrt{x+9}+3}{\sqrt{x}} \\ &x(y+1)^2 = 36\left(y^3 + \frac{16}{9}\right) \end{aligned}\right\}$$

3. $$x^2 - \frac{27x}{4} + 25 = 7\sqrt{x}\,(5-x).$$

4. $$\left.\begin{aligned} &3\sqrt{3x^3 - 4y^3} = 2x^{\frac{3}{4}} + 64 - \frac{201y^2}{4} \\ &\frac{x^{\frac{3}{2}} + 6y^2}{5y} = \frac{x^{\frac{3}{2}} + 2y^2}{x^{\frac{3}{4}} + y} \end{aligned}\right\}$$

5. A person buys a quantity of corn, which he intends selling at a certain price; after he has sold half his stock, the price of corn suddenly falls 20 per cent., and by selling the remainder at this reduced price, his gain on the whole is diminished 30 per cent.; if he had sold $\frac{3}{4}$ths of his stock before

the price fell, and the diminution in the price had then been in the proportion of £20 on the prime cost of what he before sold for £100, he would have gained by the whole as many shillings as he had bushels of corn at first. Find what the corn cost him per bushel, and what he hoped to gain per cent.

6. A and B set their respective watches right, A at noon on one day, B at noon on the following day; at 12 o'clock of the third day by B's watch B has gained (a) minutes upon A, and is then as much behind A as he is before him on the 4th day at 12 o'clock by A's watch; find their respective rates of gaining (supposed uniform).

7. A and B set out at the same instant from the foot of a hill on their ascent, A proceeding in a direct line up the side whilst B travels at a given rate on a zigzag road which intersects A's path in a point D; after an interval of (n) hours B arrives at D, and when (m) hours have elapsed from setting out, A meets C descending at a given rate by another zigzag road which crosses A's path in two points E and F; now had A waited at D till B arrived there and then proceeded as before, he would have met C at the point F and reached the summit (p) hours after doing so, and (q) hours after leaving B. Required A's rate of travelling, and the height of the hill; assuming that the roads by which A, B and C travel have an ascent of one in r, s, and t feet respectively.

1841.

1. $$\sqrt{\frac{x}{4}+3}-\sqrt{\frac{x}{4}-3}=\sqrt{\frac{2x}{3}}.$$

2. $$\left.\begin{aligned} xz &= y^2 \\ (x+y).(z-x-y) &= 3 \\ (x+y+z).(z-x-y) &= 7 \end{aligned}\right\}$$

3. $x^2(x^2 - 23) = 10x(x^2 - 24) + 649.$

4. $$\left.\begin{array}{r} x^2 + y^2 = b^2 - a^2 \\ 2x\left(x + \sqrt{x^2 + a^2}\right) + \sqrt{b^2 - y^2}\left(2y - \sqrt{b^2 - y^2}\right) = y^2 - a^2 \end{array}\right\}$$

5. A rectangular parallelopiped of stone, containing twelve cubic feet, is cut into three other parallelopipeds also rectangular, of which the smallest is a cube. The largest and smallest together contain five times as many cubic feet as the other, and the largest has its two greatest faces square. Find the dimensions of the stone.

6. The shares of a certain company are of two descriptions, *A* and *B*, the holders of which are entitled to one and two votes respectively. At an election of a director, the number of voters from *A* exceeds the number of voters for the defeated candidate from *B*, by one-ninth of all the votes given; the successful candidate polling as many men from *A*, as the other does from *B*. The whole number of votes from the class *B* is 550; and it is observed that if the defeated candidate had polled as many more votes as his opponent had voters, the whole number of votes given remaining as before, he would have been at the head of the poll with a majority less by 200 than that of the present successful candidate. Determine the number of votes given for each candidate from each class.

7. Two pieces of artillery, *B* and *C*, distant from each other 2145 yards, are moved towards each other at the same rate, *B* firing once at *C*, and *C* twice at *B*, in each minute; and each fires before starting. *C*'s twelfth report is heard at *A*, a station in *CB* produced, one minute later than *B*'s sixth report is heard at *C*, and half a minute later than it is heard at *B*. Find the rates of *B* and *C*, and the distance of *A* from the initial position of *B*. Sound travels at the rate of 1090 feet per second in still air, and the wind is moving from *B* towards *C* at the rate of 50 feet per second.

1842.

1. $$\frac{x - 4\frac{2}{3}}{3} - \frac{2x - 3\frac{2}{3}}{4} = \frac{3}{2}\left\{x - \frac{x - 1\frac{1}{2}}{2}\right\} + \frac{4x}{3}\left\{x - 3 - \frac{(x-1)(x-2)}{x}\right\}.$$

2. $$\left.\begin{aligned} &\frac{2}{3}\left\{x - \frac{3}{5}y\right\} + \frac{x + \frac{y}{5}}{6} = \frac{1}{3} - \frac{1}{2}\left\{\frac{\frac{4}{5}y - 2}{6} - (x - y)\right\} \\ &x - 2y - \frac{3y - 5x}{2} = \frac{11}{2}(x + y) + 3(x - y) \end{aligned}\right\}$$

3. $$\left(x - \frac{1}{3}\right)^2 - \frac{25}{9} = \frac{3x^2 + \frac{4}{9}}{2\left(x - \frac{1}{3}\right) + \sqrt{x\left(x - \frac{8}{3}\right)}}.$$

4. $$\left.\begin{aligned} &\frac{3 + 2x^2 - 4x^4}{x^2 - 1} = y^2(1 - 2y^2) \\ &(2x^2 - 1)(2y^2 - 1) = 3. \end{aligned}\right\}$$

5. A person leaves London for Derby by the Birmingham railway at 10 A.M., intending to get upon the Midland Counties line at Rugby, and allowing for a delay of 30′ in changing trains, but expecting to travel the 48 miles from thence to Derby at the same rate at which he had come down, he calculated to reach Derby at 4 P.M. On reaching Rugby, however, he finds that there will be no train for Derby till too late for his purpose: but that by going on upon the first line to Hampton ($\frac{1}{4}$ as far again as he had come already) he might start immediately by the Derby Junction line, and though the whole distance by this route would be 13 miles longer than by the other, yet the speed on the second line being one mile an hour quicker than on the other, he would reach Derby (supposing

time were accurately kept upon the road) just $1\frac{1}{2}'$ before 4. What is the distance from London to Rugby?

6. Two cubical boxes A, B, of which B is larger by 1216 cubic inches, are filled with balls, there being 12 more along an edge of B than in an edge of A, and the number of balls in the faces of A is to the number in the edges of B as 7 : 22. Also the difference between the areas enclosed by the balls of B (defined by a thread passing round them) when they are spread out first into a hollow and then into a solid square, is to the same difference with respect to the balls of A as $129\frac{181}{900} : 1$. Find the radii of the balls.

7. The income of a schoolmaster arises partly from ten pupils residing in his house; and partly from an endowment, which produces a certain number of quarters of wheat each year. When wheat sells for 60*s*. the expenditure of his family (£249) is less than his savings by a number, which, when divided by twice the number of his pupils, expresses the proportion which the clear gain bears to the whole charge for each pupil. In the following year wheat falls to 55*s*. and a tax of 8*d*. in the pound is laid upon income, payable upon the net income of the preceding year; but the cost of living for his pupils being diminished, (so that, in fact, the amount of income-tax he has to pay, with 10*s*. added, would just support one pupil,) he finds that his savings are greater than in the year previous by a sum equal to the difference of his net income in the two years, which is $\frac{1}{18}$th of the expenditure of his family in the second year, besides allowing for an outlay of £15 in repairs. The net income from pupils in the first year being £330, find that from the endowment in the same year, and the ratio of the costs of living in the two years.

1843.

1. $$\frac{1}{x^2+11x-8}+\frac{1}{x^2+2x-8}+\frac{1}{x^2-13x-8}=0.$$

2. $$\left.\begin{aligned}(x-2)^2+(y-3)^2+(z-1)^2&=24\\ xy+xz+yz&=63\\ 2x+3y+z&=30\end{aligned}\right\}$$

3. $$\frac{(n-1)(a^4+a^2x^2+x^4)}{(n+1)(a^4-a^2x^2+x^4)}=\left(2-\frac{1}{n}\right)\left(\frac{ax}{a^2-x^2}\right)^2.$$

4. $$\left.\begin{aligned}&(x^4+2bx^2y+a^2y^2)(y^4+2bxy^2+a^2x^2)\\ &\quad=4(a^2-b^2)(b+c)^2(xy)^2\\ &\quad x^3+y^3=2cxy\end{aligned}\right\}$$

5. A merchant travelling from St. Petersburgh to Moscow had provided himself with notes of the bank of Russia, amounting in all to 540 rubles. During the first part of the journey the paper bore the value marked on it, but in all places south of Torjok, a town on the road, as well as in Moscow itself, a premium of 20 per cent. was allowed on each note. On arriving in Moscow the merchant received 432 rubles for the notes that remained: he spent 237 rubles during his stay; and he had exactly enough left to pay his expenses back, supposing them to be the same as in his journey thither. How many rubles had he spent between St. Petersburgh and Torjok, and how many between Torjok and Moscow?

6. From a quantity of gold, silver, and copper, weighing in all 20,300 oz. two alloys were formed. In the one gold and copper were mixed in the proportion of 11 : 1; in the other silver and copper in the proportion of 37 : 3; and there were 288 oz. of copper over. The alloy of gold and copper was coined at the rate of £3 17*s.* 10½*d.* per ounce, and the alloy of silver and copper at the rate of 5*s.* 6*d.* per oz. The whole sum thus produced was £5,546 14*s.* 6*d.* What were the quantities of each metal?

7. A body of 6,048 soldiers was divided into a number of equal detachments and sent to occupy as many fortresses. In the course of the campaign as many as two whole garrisons and half of another died in an epidemic, and all the rest, except 84 invalids, who returned to head quarters, were equally divided among the fortresses as before. But the reduced garrisons proving too weak for their defence, all the fortresses fell into the hands of the enemy, and the men, with the exception of four whole garrisons and 210 fugitives were killed or taken prisoners. The loss thus sustained, together with that occasioned by the epidemic, amounted to 4,186 men. Required the number of fortresses, and the number of men sent at first to occupy each.

1844.

1. $\dfrac{4x-17}{x-4}+\dfrac{10x-13}{2x-3}=\dfrac{8x-30}{2x-7}+\dfrac{5x-4}{x-1}.$

2. $(x^2-5)^2=(x-3)^2+(x+1)^2.$

3. $$\left.\begin{aligned} xy(y+x-z) &= \tfrac{3}{4} \\ yz(z+y-x) &= 7\tfrac{1}{2} \\ zx(x+z-y) &= 3 \end{aligned}\right\}.$$

4. $$\left.\begin{aligned} (x^2+2xy)(x^2+8) &= 49+8xy \\ (4x^2-4xy)(xy-3x^2-2) &= 3(x^2+1)^2 \end{aligned}\right\}$$

5. An author calculates on printing a work for a certain sum, reckoning that $1\frac{1}{2}$ of his MS. pages go to a printed one. The first half of the MS. accords with this supposition: but having in the latter half increased the width of his writing, $5\frac{5}{8}$ of his pages are now equivalent to 4 of his first, and $\frac{1}{3}$ of a printed page. The real cost thus falls short of his estimate by £4 8*s*. $10\frac{2}{3}$*d*. Had he written $\frac{2}{3}$ of his MS. before

changing, his estimate would have exceeded the real cost by a number of pounds less by $37\frac{1}{27}$ than $\frac{1}{5}$th the difference between the actual number of pages of MS. and the number of printed pages. Find the number of pages of MS. and the amount of his estimate.

6. There are 500 tickets on sale for a public spectacle, divided equally into two classes A and B, the price of the former being double that of the latter. After five times as many of B are sold as of A, the proprietor raises the price of B in the ratio $3 : 4$, and lowers those of A in the ratio $6 : 5$. The ratio of the sums taken after and before the change is less than half the number of tickets A left, by $1\frac{1}{7}$. All the tickets B are sold except 10 : and the excess of tickets A sold since the change, over what there is left of that class, is greater than the whole number of class B sold since the change, by the excess of the number of tickets A unsold over the number of tickets B unsold. Find the number of tickets sold of class B more than of class A.

7. Two men, A and B, engage to mow a piece of grass in 9 hours, their rates of working being as $2 : 3$. They mow together for a certain time, and after that they work separately for equal times, B commencing and A finishing it at double his former pace. And the quantity B did when he worked alone, exceeded by 300 square yards what he had done more than A at the end of 8 hours, and exceeded the quantity done by A when they worked together, by a number of square yards equal to 50 times the whole time occupied. Had they worked always together as they did at first, they would have finished it in $1\frac{1}{2}$ hours less than half the sum of the number of hours they agreed to, and the number they took. Find the number of hours they worked together and the time they took.

1845.

1. $\sqrt{(x-1)(x-2)} + \sqrt{(x-3)(x-4)} = \sqrt{2}$.

2. $x(\sqrt{x}+1)^2 = 102(x+\sqrt{x}) - 2576$.

3. $\left(\dfrac{2x+3}{2x-3}\right)^{\frac{1}{3}} + \left(\dfrac{2x-3}{2x+3}\right)^{\frac{1}{3}} = \dfrac{8}{13}\cdot\dfrac{4x^2+9}{4x^2-9}$.

4. $\left.\begin{array}{l} a^3 - x^2 = 3xy \\ (\sqrt{y}-\sqrt{x})(a-x) = 3\sqrt{x}(x+y) \end{array}\right\}$

5. *A* and *B* embark in trade for 5 years; *B* is to have seven-sixteenths of the net annual profits for the first half of the time, and half of them for the remainder; after $3\frac{1}{2}$ years the annual profits, by a lowering of the tariff, were increased in the proportion of 6 to 5, and at the same time became liable to a reduction of 7*d.* in the pound by the laying on of the Income-tax. At the termination of the partnership, *B*'s share of the total net profits amounted to £987; required the annual profits of the business before the duties were reduced.

6. A cubical vessel is filled with water and has its surface exposed to the sky. The temperature of the atmosphere is 30° on the first day, and every successive day it is increased by 1°. Suppose that a temperature of 15° would evaporate 1 inch in one day, and other temperatures in the same proportion. Every evening there are showers; on the first evening 3 inches of rain fall, and the depth of rain falling each successive evening decreases in an Arithmetic Progression whose common difference is $\frac{1}{90}$th of what fell on the first day. At the end of 41 days the vessel is found to be empty; required its content.

7. *A* and *B* set out together, and walk from Keswick over Helvellyn to Ambleside. *B* arrives at the foot of Helvellyn which is 5 miles from Keswick in an hour, and then slackens his pace so that he just reaches the top at the end of the 2nd hour. Here he sits down to rest, until *A* passes him and gets

as much before him (B) as he was behind when B first sat down. B then starts at an increased pace, and passes A at the end of the 3rd hour from the time of starting. B walks on at the same pace for another hour, and then waits for A to come up, whose distance behind him was $\frac{2}{3}$ of what it was at the end of 1st hour. When A overtakes him, he starts again and walks at the same pace until he reaches Ambleside; the time of this last stage being to that of his first rest in the ratio of $20:7$. On his arrival, he fires a pistol for the information of A, who having hitherto kept up a uniform pace without stopping, now diminishes it in the ratio of $5:7$, and reaches Ambleside 10 minutes after B. What is the distance from Keswick to Ambleside by this route?

1846.

1. $$\sqrt{\frac{1}{2}x+2}-\sqrt{\frac{1}{2}x-2}=\sqrt{x+3}-\sqrt{x-3}.$$

2. $$x-\frac{6}{\sqrt{x}}=5\left(1+\frac{1}{x}\right).$$

3. $$x^2(x-4)=2(x-2)(2+\sqrt{2}).$$

4. $$\left.\begin{aligned} a^2(y^2-ax)+b^2(x^2-by) &= ab\left\{y(a-x)+x(b-y)\right\} \\ (ax+by)^3 &= (a^2+b^2)\left\{ab(x^2+y^2)+xy(a^2+b^2)\right\} \end{aligned}\right\}$$

5. The number of years intervening between Alexander's expedition to the Sutlej and the late great battle there, is expressed by a certain multiple of ten, the sum of whose digits is equal to 10; the digit in the last place is twice the digit in the third place, and three times the digit in the second place is equal to the number of complete centuries that have elapsed. How many years does the interval consist of?

6. The Provisional Committee of a Bubble Company consists of 20 persons, who, after allotting a certain number of

shares, reserve the rest to be equally divided among themselves; and each sells 50 at a certain premium, and afterwards a further number at only half the former premium, until he has only 100 shares left. It is then resolved to wind up; and the expenses, amounting to 10 shillings a share on the whole Stock of the Company, are obliged to be paid by the Committee, who find that they have neither lost nor gained by the transaction. They now commence actions against the Allottees (all of whom, like the Committee, had neglected to pay their Deposits), and recover 10 shillings a share from each, the law expenses (defrayed by the Committee) amounting to a fourth of the sum recovered; and their gain is now $\frac{1}{8}$th of what it would have been, if they could have sold all their shares at the original premium. On a new trial, the judgment is reversed (the same law expenses as before being incurred, and defrayed by the Committee), and the entire loss of the Committee is now £400 more than the gain which each expected to make by selling all his shares at the original premium. Find the number of shares allotted, and the number reserved.

7. To accommodate several factories along a stream, in the dry season, a reservoir is made, in form a parallelopiped with sides in Arithmetical Progression, which when full will supply them for 66 days with a cube of water per day whose side is equal to the shortest edge of the parallelopiped, which edge in yards is equal to twice the number of factories. The cost is defrayed by a tax on the factories proportional to the water each receives, which in consequence of part of it being diverted as it runs down the stream for the successive factories, is in Arithmetical Progression whose common difference is $\frac{1}{13}$ of the quantity received by the first. The cost of making it at 3*d.* per cubic yard : tax on the last factory :: eleven times the number of factories : 9. Required the sums paid by the first and last factories.

1847.

1. $\frac{2}{19}\left(\sqrt{x^2+39x+374}-\sqrt{x^2+20x+51}\right)=\sqrt{\frac{x+22}{x+17}}.$

2. $$\left.\begin{aligned}(v+x)(y+z)&=b+c-a\\(v+y)(x+z)&=a+c-b\\(v+z)(x+y)&=a+b-c\\v^2+x^2+y^2+z^2&=3(a+b+c)\end{aligned}\right\}$$

3. $$\left.\begin{aligned}x^2y^2\left\{\sqrt{xy+4}-2\sqrt{2}\right\}&=(4-xy)\sqrt{2}\\\sqrt[3]{x+2}-\sqrt[3]{y+2}&=y-\frac{4}{y}\end{aligned}\right\}$$

4. $(1-x)\sqrt{a\left(1+\frac{1}{x}\right)-2}=\sqrt{x+1}+\sqrt{3x-1}.$

5. Three horses A, B, C start for a race on a course a mile and a half long. When B has gone half a mile, he is three times as far a head of A as he is of C. The horses now going at uniform speeds till B is within a quarter of a mile of the winning post, C is at that time as much behind A as A is behind B, but the distance between A and B is only $\frac{1}{11}$th of what it was after B had gone the first half mile. C now increases his pace by $\frac{1}{53}$rd of what it was before, and passes B 176 yards from the winning post, the respective speeds of A and B remaining unaltered. What was the distance between A and C at the end of the race?

6. A and B set out to walk together in the same direction round a field which is a mile in circumference, A walking faster than B. Twelve minutes after A has passed B for the third time A turns and walks in the opposite direction until six minutes after he has met him for the third time, when he returns to his original direction and overtakes B four times more. The whole time since they started is three hours, and A has walked eight miles more than B. A and B diminish their rate of

walking by one mile an hour at the end of one and two hours respectively. Determine the velocities with which they began to walk.

7. At the late election of a Chancellor the number of voters at St John's and Trinity in favour of Lord Powis was less than ten times the number of those at St John's who voted for Prince Albert by twice the first digit in this number, and was also 40 times the sum of the digits in the whole number of voters at Trinity. The sum of the digits in the number of voters at St John's for Lord Powis was a mean proportional between the corresponding sums in the voters at St John's and Trinity for the Prince. The majority for Lord Powis at St. John's and Trinity together increased by unity was to the sum of the last two digits of the number of Trinity voters for the Prince as its excess over the number expressed by these two digits was to the excess of the first digit of the voters at St. John's for the Prince over the second; and each of these ratios was equal to that of the number of voters at St John's for Lord Powis, to the number of voters for the Prince *i. e.* $= 6 : 1$. The last digit of the number of voters at St. John's and the first of those at Trinity for the Prince are the same as the first digit of the number of voters at St. John's for Lord Powis; and the last digit of this number together with the middle digit of the whole number of voters at Trinity equals the last digit of the voters at Trinity for the Prince, together with the sum of the digits of the voters at St. John's for him. The second and first digits in the voters at St. John's for Lord Powis; the first in those at St. John's, and the middle in those at Trinity, for the Prince, are in Arithmetical Progression. Less than 100 at St. John's voted for the Prince.

Find the number of voters at St John's and Trinity for each of the Candidates.

1848.

1. $$\frac{\frac{3}{x}-1}{2}-\frac{9\left(\frac{1}{2x}-1\right)-\frac{2}{5}\left(\frac{9}{2x}-4\right)}{\frac{3}{x}-4}=\frac{\frac{9}{x}+19}{6}.$$

2. $$\sqrt{a+x}+\sqrt{a-x}=\sqrt{\frac{3b^2+x^2}{a+b}}.$$

3. $$\left.\begin{aligned} x^3 &= 31x^2-4y^2 \\ y^3 &= 31y^2-4x^2 \end{aligned}\right\}$$

4. $$\left.\begin{aligned} (x+y)^{\frac{1}{3}}+(x-y)^{\frac{1}{3}} &= a^{\frac{1}{3}} \\ (x^2+y^2)^{\frac{1}{3}}+(x^2-y^2)^{\frac{1}{3}} &= a^{\frac{2}{3}} \end{aligned}\right\}$$

5. A railway train travels from A to C passing through B where it stops 7 minutes; two minutes after leaving B it meets an express train which started from C when the former was 28 miles on the other side of B: the express travels at double the rate of the other, and performs the journey from C to B in $1\frac{1}{2}$ hours; and if on reaching A it returned at once to C it would arrive 3 minutes after the first train. Find the distances between A, B, and C and the speed of each train.

6. To meet a deficiency of (m) millions in the revenue of a country, an *additional* tax of (a) per cent. was laid upon articles exported, and the tax upon imports was diminished (c) per cent.: in consequence of these alterations the value of the imports was increased so as to be (n) times as great as the exports, and the deficiency was made up. It was afterwards found that if the additional tax upon the exports had been (a') per cent., and the tax upon imports diminished (c') per cent., the values of the articles being altered as before, the deficiency would not have been made up by (m') millions. Find the values of the exports and imports after the alteration of the tax.

7. Fifty thousand voters who have to return a member to

an assembly, are divided into sections of equal size, and each section chooses an elector, the member being returned by the majority of such electors. There are two candidates A and B. In those sections which return electors favourable to A, the majority is double the minority, while in those favourable to B, the minority forms only a tenth of the whole. After the primary elections a third candidate C comes forward, and is joined by so many electors of each party, that he is returned by a majority of 3 over A, and 14 over B. If C had not come forward, A would have been returned by a majority 19 less than the whole number of votes actually polled by C, and if the elections had been by the 50,000 voters *directly* between A and B, B would have had a majority of 6000. Find the number of sections.

1849.

1. $2b\left\{\sqrt{x+a}-b\right\}+2c\left\{\sqrt{x-a}+c\right\}=a.$

2. $\sqrt{2x-1}+\sqrt{3x-2}=\sqrt{4x-3}+\sqrt{5x-4}.$

3. $8x^{\frac{3}{4}}+81=18x^{\frac{1}{2}}+45x^{\frac{1}{4}}.$

4. $$\left.\begin{aligned}\frac{y-x+\sqrt{2xy-3x^2}}{y-2x}&=3\,\frac{(2y-3x)^{\frac{3}{2}}+x^{\frac{3}{2}}}{(2y-3x)^{\frac{3}{2}}-x^{\frac{3}{2}}}\\ \frac{y}{x^2}+\frac{16}{81}\left(x-\sqrt{x}-\frac{3}{4}\right)&=\frac{4}{9x}(2\sqrt{xy}-\sqrt{y})\end{aligned}\right\}$$

5. The distance between the two termini A and Z of a railway is 100 miles. A train starting from A runs up-hill during the first 30 miles of its journey, the next 50 miles are on a level, and the remaining 20 are up-hill. The train may be supposed to travel 5 miles an hour faster on the horizontal road than when it is ascending a hill. There are to be stoppages B, C, D, and E, at distances 20, $42\frac{1}{2}$, $67\frac{1}{2}$, and 90 miles respec-

tively from A, and each stoppage may be supposed to cause a detention of 3 minutes. Find the time of arrival at B, C, D and E, of the train which starts from A at $8^h. 0^m.$ and arrives at Z at $12^h. 42^m.$

6. A number of vessels $A_1 A_2 A_3 \ldots A_r A_{r+1} \ldots A_m$ are arranged in a row. A_1 contains a quantity of wine, A_2 a quantity of water, and the remaining vessels $A_3 \ldots A_r A_{r+1} \ldots A_m$ contain any quantity of any other fluids. 1-n^{th} part of the wine in A_1 is taken from A_1 and added to the contents of A_2, 1-n^{th} of the mixture is taken from A_2 and poured into A_3, 1-n^{th} of the contents of A_3 is poured into A_4, and so on to the end of the series of vessels. Again, 1-n^{th} of wine remaining in A_1 is poured into A_2, 1-n^{th} of the contents of A_2 into A_3 and so on. A_1 is supposed never to receive any addition. It is found that 60 times the quantity of wine in the vessel A_r after $r-1$ abstractions of fluid from that vessel $= 31$ times the quantity of wine in the same vessel after r abstractions. Also 59 times the quantity of water in A_r after $r-1$ abstractions of fluid from that vessel $= 31$ times the quantity of water in the same vessel after r abstractions. Find the numerical values of r and n.

7. Two points P and Q are connected by a wire (A), $\frac{1}{8}$th of an inch in diameter and 50 miles in length, which is used for transmitting a galvanic current. It is required to replace the wire (A) by three others (a), (b), and (c), composed of different metals and of lengths 50, 60 and 70 miles respectively. These new wires must be of such diameters that the current, which previously passed along (A), may be divided so that the quantities which pass along (a), (b), and (c) may be as 3, 4, and 5. The quantity of galvanic fluid that will pass along a wire is supposed to vary inversely as the resistance, and the resistance to vary directly as the length of wire to be traversed, inversely as the sectional area of the wire, and inversely as the conductibility. Also the sectional area of a wire varies as the square of its diameter. It is found by experiment that the quantity of

galvanic fluid which will pass along a portion of the wire (A), $\frac{1}{8}$th of an inch in diameter and 15 yds. long, may be denoted by 1000 k. Also portions of wire $\frac{1}{10}$th of an inch in diameter, composed of the same metals as (a), (b), and (c) of lengths 20, 10, and 40 yds. respectively, are capable of transmitting quantities of galvanic fluid 750 k, 5400 k, and 3500 k respectively. Find the least possible diameters of wires (a), (b), and (c) in order that the above conditions may be satisfied.

1850.

1. $$(x + a)\left(1 + \frac{1}{x^2 + a^2}\right) + \sqrt{2ax}\left(1 - \frac{1}{x^2 + a^2}\right) = 2.$$

2. $$\frac{1}{6x^2 - 7x + 2} + \frac{1}{12x^2 - 17x + 6} = 8x^2 - 6x + 1.$$

3. $$\left.\begin{aligned} (x^2 + y^2 + c^2)^{\frac{1}{3}} + (x - y + c)^{\frac{2}{3}} &= 2\,(4xy)^{\frac{1}{3}} \\ \frac{1}{y} &= \frac{1}{x} + \frac{1}{c} \end{aligned}\right\}$$

4. $$\left.\begin{aligned} 2(x^2 + xy + y^2 - a^2) + \sqrt{3}\,(x^2 - y^2) &= 0 \\ 2\,(x^2 - xz + z^2 - b^2) + \sqrt{3}\,(x^2 - z^2) &= 0 \\ y^3 - c^3 + 3\,(yz^2 - c^3) &= 0 \end{aligned}\right\}$$

5. A lends one half of his money to B at 5 per cent. per annum simple interest, and the remaining half he invests in the three per cents. at 90. B pays the interest regularly during the first five years, but afterwards neglects to do so till other five years' interest is due when A calls in all his money, and B becomes a bankrupt paying 10*s*. in the pound. A sells out when the funds are at 81, and then he finds that the whole sum he has received as principal and interest in the ten years exceeds the sum that he originally possessed by £34 13*s*. 4*d*. How much did he lend B?

6. A, B and C are three villages. The road from A to B is level, and C is on a hill above A and B. The distances AB, BC and CA are respectively 24, 14.4 and 28.8 miles. P walks up hill $\frac{1}{4}$th slower and down hill $\frac{1}{3}$rd faster than when the road is level. Q walks up hill $\frac{1}{5}$th slower and down hill $\frac{1}{5}$th faster than when the road is level. P travels round in the direction ACB in 30 minutes less than Q requires to go round in the opposite direction. Q travels round in the direction ACB in 1 hour 48 minutes more than P takes to make the circuit in the opposite direction; find the rates of each on a level road. Also supposing them both to start from A in opposite directions, find their points of meeting.

7. Suppose that in the course of any one year the number of births in Ireland is on an average 32 for 1000 living in the island at the commencement of that year, the number of deaths and emigrations to the colonies 21 in 1000 and of migrations to England 1 in 100. In England suppose the number of births in the course of any one year to be 3 for every 100 inhabitants living at the beginning of the year, the number of deaths and emigrations to the colonies 289 in 10,000, and of migrations to Ireland 1 in 10,000. If the number of inhabitants in England was twice the number in Ireland at the beginning of 1850, in what year will the population of the former be three times that of the latter, according to the law above stated.

1851.

1. $$\frac{x^2+2x+2}{x+1}+\frac{x^2+8x+20}{x+4}=\frac{x^2+4x+6}{x+2}+\frac{x^2+6x+12}{x+3}.$$

2. $$(5x^2+x+10)^2+(x^2+7x+1)^2=(3x^2-x+5)^2 + (4x^2+5x+8)^2.$$

3. $$(x^2+4x-2)^2+3=4x(3x^2+4).$$

4. $$\left.\begin{aligned} &3x + 3y - z = 3. \\ &x^2 + y^2 - z^2 = \frac{14 - 9z}{2}. \\ &x^3 + y^3 + z^3 = 3xyz + \frac{17z + 44}{4}. \end{aligned}\right\}$$

5. A derives his income from a fixed rental, B from his profession, C from both. In the first year A pays as much income-tax as B and C together, but in the second year B's and C's professional incomes being doubled, B pays as much as C, which is $\frac{4}{5}$ths of what A pays; also the total amount of their incomes in the two years is £5500. Assuming that the income-tax is higher for a fixed rental than for professional income in the ratio 3 : 2, find the incomes of A, B, C in the first year.

6. A and B start at the same time in a boat-race; A has 100 yards start and has to row to a post D, B to a post C. At first A's rate : B's :: 40 : 39, but when the distance between A and B is $\frac{1}{6}$th of the remaining distance A has to row, A's speed is diminished in the ratio 79 : 80, so that two minutes afterwards the distance between them is three yards more than half the remaining distance B has to row. At this point of B's course, his rate which has hitherto been uniform is increased eight yards a minute, while A's is still further diminished six yards a minute; and in one minute more B arrives at the post C, A being then three yards from the post D. Find the distance between C and D.

7. Three men, A, B, C walk in the same direction in the circumferences of three concentric circles, starting simultaneously from points where they are at their least distances from each other. A walks his circuit in an even number of hours, (greater than four), B and C their circuits in one hour and two hours less respectively. Whenever A and B are at their greatest distance from each other, they alter their rates in such a manner, that the times they would take to walk their circuits

at the rates they are *then* going are interchanged; and whenever A and C are *again* at their least distance their times are interchanged in a similar manner. When A and B are at their greatest distance the first time, A has walked a distance equal twenty-two times C's circuit; and when they are at their greatest distance the third time, B has walked a distance equal forty-two times A's circuit, and C has then walked ten miles less than forty times B's circuit and is at his least distance from B. Required the rates of A, B, C at first.

ANSWERS.

*** If in the irrational parts of an equation values deduced from the following results be substituted, due attention must be paid to the signs of those values. Thus, suppose $x = 4$, then $\sqrt{x} = \pm 2$, and actual substitution alone will determine the proper sign. Also, the values of xy....are arranged in pairs, and the signs must be taken in order; thus the first value of x with upper sign (if there be two signs) corresponds to the first value of y with upper sign, and so on.

1794. 1. 6. 2. $x = 5, y = 6$.

3. $x = 5, \frac{1}{5}, -\frac{17 \pm 6\sqrt{-2}}{19}$; $y = 3, -15, -(6 \pm \sqrt{-2})$.

4. 27, 81. 5. 45 turkeys, 60 geese.

6. *A*'s 3*s*., *B*'s 2*s*. per day.

1795. 1. 9. 2. $x = 7, y = 10$. 3. $4, \frac{2}{5}$.

4. 4550. 5. 100 crowns, 40 pieces, 860 guineas.

6. 3 men, 2 boys.

1796. 1. 24. 2. $x = 2, y = 4$. 3. $4, -1$.

4. *A*'s = 24, *B*'s = 36.

5. £100 at 4 per cent., £400 at 2 per cent.

6. Breadth of walk 3 yards; side of court 16 yards; area of court 256 yards.

1797. 1. 4. 2. $x = 5, y = 6$. 3. $3, \frac{41}{2}$.

4. For *A* and *C*, 60; for *A* and *B*, 72; for *B* and *C*, 30.

5. Circumferences of wheels are 20 and 12 yards; lengths of strings 360 and 72 yards.

6. No. of acres in *A*'s 12; no. in *B*'s 8.

1798. 1. 3. 2. $x = 15, y = 3.$ 3. $4, -\frac{29}{9}.$

4. $x = \pm 3, y = 2;\ x = \pm\sqrt{-6}, y = -1;$

$$x = \pm\sqrt{\frac{15 \pm \sqrt{45}}{2}},\ y = \frac{1 \pm \sqrt{45}}{2};$$

$$x = \pm\sqrt{\frac{15 \pm \sqrt{-47}}{2}},\ y = \frac{1 \pm \sqrt{-47}}{2};$$

$$x = \pm\sqrt{\frac{15 \pm \sqrt{-11}}{2}},\ y = \frac{1 \pm \sqrt{-11}}{2}.$$

5. 820 men. 6. 300 yards. 7. 2, 4, 6, 8 days.

1799. 1. 5. 2. $x = 7, y = 2.$ 3. $3, \frac{21}{11}.$

4. $x = 3, -\frac{135}{46};\ y = 2, -\frac{115}{79}.$ 5. £4 11*s*.

6. 24 bales or 72 casks. 7. 36 and 16 square yards.

1800. 1. 20. 2. $x = 2, y = 3.$ 3. $3, -8.$

4. $\pm 2, \pm\sqrt{-720796}.$ 5. 10 to each.

6. True time $8^h\ 45'$. 7. $N = 5.$

1801. 1. $\dfrac{1}{1-a}.$ 2. $x = 12, y = 7.$ 3. $9, -13.$

4. $2, -\frac{8}{7}.$ 5. £16. 6. 19.

7. *A* won 8 and *B* 28 games.

1802. 1. 4. 2. $x = 21, y = 20.$ 3. $7, \frac{3}{2}.$

4. $5, -6.$ 5. 13 shills. 6. 4 and 5 yards.

7. Area 48 sq. yds., sides 6 and 8.

1803. 1. 7. 2. $x = 8, y = 9.$ 3. $19, -19\frac{2}{3}.$

4. $x = \frac{9}{4}, y = 16.$ 5. 60.

6. 3 per cents. 60, 4 per cents. 75.

7. No. of merchants 25; pay of captain £500.

1804. 1. 21. 2. $x = 10, y = 15.$ 3. $\frac{23}{2}, 4.$

4. $x = 4, \frac{144}{49}, \frac{81}{16}, \frac{729}{196}, y = 9, \frac{49}{4}, \frac{784}{81}, \frac{9604}{729}.$ 5. 936.

6. A loaf cost 7*d*.; a bottle of wine $11\frac{1}{2}$*d*. 7. 7.

1805. 1. 8. 2. $x = 3, y = 2.$

3. $3, -\frac{9}{2}, \dfrac{-3 \pm \sqrt{-55}}{4}.$

4. $x = 9,\ 4,\ \dfrac{-13 \pm \sqrt{-51}}{2},$

$y = 4,\ 9,\ \dfrac{-13 \mp \sqrt{-51}}{2}.$

5. 125, 343 cubic feet. 6. 9 yds. of better, 7 of worse.

7. 16.

1806. 1. 11. 2. $x = 3,\ y = 5.$ 3. $a,\ \dfrac{b^2}{n^2 a}.$

4. $x = \pm 20,\ \pm 16;\ y = \pm 16,\ \pm 20.$ 5. 256 gallons.

6. 27. 7. *B* 15 days, *C* 18 days.

1807. 1. 7. 2. $x = 8,\ y = 13.$ 3. $9,\ \frac{71}{5}.$

4. $1, \left(-\dfrac{8}{7}\right)^{\frac{3}{2}}.$ 5. 6*d*.

6. No. of guineas 80; of shillings 20; value of an adulterated guinea 19*s*.; of an adulterated shilling 9*d*.

7. After 6 days.

1808. 1. 10. 2. $x = 2,\ y = 6.$ 3. $3,\ \frac{6}{5}.$

4. $4,\ -2;\ -1 \pm \sqrt{-3}.$ 5. 20. 6. 25 miles.

7. Area of base 36 feet.

1809. 1. 8. 2. $x = 10,\ y = 5.$

3. $x = 3,\ -\frac{11}{3};\ y = 1,\ -\frac{7}{3}.$ 4. $4,\ 1,\ \left(\dfrac{1 \pm \sqrt{-11}}{2}\right)^2.$

5. The first discharged 3, the second 2 gallons, and 14 galls. flowed through pipe in 1′.

6. Area of △ 6, of rectangle 10; sides of △, 3, 4, 5; sides of rectangle 2, 5.

7. No. of balls in 1st pile 56; in 2nd 50; no. of layers in 1st pile 6.

1810. 1. 9. 2. $x = 4,\ y = 3.$ 3. $2,\ -3\frac{2}{3}.$

4. $x = \frac{1}{9},\ 4,\ 9,\ 1;\ y = \frac{16}{9},\ 4,\ 0,\ 4.$ 5. £1000.

6. No. of men 16, of women 20, of children 32; each man had 3*s*., each woman 2*s*. 6*d*., each child 2*s*.

7. Diagonal, 36, common ratio, 81.

1811. 1. 7. 2. $x = 4, y = 3$. 3. $4, -\frac{65}{6}$.

4. $x = \left(-4 \pm 2\sqrt{\frac{-13}{3}}\right)^2, y = 1 \pm 2\sqrt{\frac{-13}{3}}$.

$x = 4, \frac{4}{9}, \frac{4}{25}, 4;\ y = 3, \frac{5}{3}, \frac{7}{5}, -1;$

$x = \frac{1}{25}(12 \pm \sqrt{644})^2, y = \frac{1}{5}(37 \pm \sqrt{644})$.

5. 20 Farmers. 6. 693. 7. 1, 2, 4.

1812. 1. 5. 2. $x = 7, y = 4$. 3. $3, -\frac{9}{4}$.

4. $\sqrt{x} = \pm 9, \pm 4;\ \sqrt{y} = \pm 4, \pm 9$.

$\sqrt{x} = \pm \frac{9\sqrt{-1} \pm 2\sqrt{22}}{\sqrt{3}}, \sqrt{y} = \mp \frac{9\sqrt{-1} \mp 2\sqrt{22}}{\sqrt{3}}$.

5. 240 apples; 60 oranges, which are worth $1\frac{1}{2}d.$ each.

6. Bushel of oats $5s.$; basket of turf $4d.$

1813. 1. 9. 2. $x = 1, y = 4$. 3. $3, -3$.

4. $x = 4, \frac{1}{4};\ y = 2, -3$.

$x = \left(\frac{-1 \pm \sqrt{17}}{4}\right)^2, y = \frac{-5 \pm \sqrt{17}}{2}$.

5. 900 men. 6. *A* worked 14 days; 49 days.

7. Height 30 feet, length 25 feet.

1814. 1. 35. 2. $x = 9, y = 2$. 3. $3, -\frac{4}{5}$.

4. $x = 12, -\frac{19}{2};\ y = 2, -\frac{19}{12}$. 5. 100. 6. 26.

7. Ratio of pure gold to alloy in a guinea 11 : 1 or 239 : 11. Ratio of value of pure gold to that of an equal quantity of alloy 239 : 11 or 11 : 1.

1815. 1. 72. 2. $x = 7, y = 4$. 3. $5, 6\frac{9}{10}$.

4. $x = 5, -\frac{23}{5}, \frac{1 \pm 3\sqrt{-61}}{5};\ y = 3, -\frac{69}{25}, \frac{3}{25}(1 \pm 3\sqrt{-61})$.

5. 180000. 6. 10; £12, £17, £22, &c.

7. At the 25th milestone.

1816. 1. 51. 2. $x = 18, y = 24$. 3. $6, \frac{40}{13}$.

4. $x = 4, 0, \sqrt[3]{18\sqrt{-1}};\ y = 25, 9, -9$.

5. Rates of sailing 11, 5, 7 miles an hour; distance 22 miles.
6. Lengths of streets 18 and 30 chains; of sewer 21; distance of mouth from *B*, 10 chains.
7. Family at first 10; labourer saved 4*s*.; each member saving 3*d*. less than preceding. Price of wheat 8*s*. per bushel.

1817. 1. 3. 2. $x = \frac{4}{5}a,\ y = \frac{3}{4}a$.
3. $x = 4;\ \sqrt{x} = -\frac{1}{3}$. 4. $x = \frac{3}{4},\ y = \pm 2$.
5. 40 bushels at 10*s*. per bushel. 6. 79 days, 28 men.
7. Quantity in hold 1200 gallons.
Horary influx 120 gallons.

1818. 1. 7. 2. $x = 5,\ y = 6$. 3. $4,\ (-7)^{\frac{2}{3}}$.
4. $x = \pm 9,\ \pm \frac{45}{4},\ \pm \frac{8\sqrt{-1}}{\sqrt{271}},\ \pm \frac{10\sqrt{-1}}{\sqrt{271}}$.

$$y = \pm 4,\ \pm \frac{16}{5},\ \pm \frac{\sqrt{-271}}{8},\ \pm \frac{\sqrt{-271}}{10}.$$

5. Born in 1742; died aged 63.
6. £700 in first and £100 in second.
7. Width of *A*'s and *B*'s large warehouse was 52 feet; of small one 20 feet; and the width of *C*'s 48 feet.

1819. 1. 5. 2. $x = 6,\ y = 5$. 3. $\frac{n}{q},\ -\frac{p}{m}$.
4. 40. 5. *A*'s stock £600, *B*'s £400.
A's gain £60, *B*'s £40.
6. *A*'s daily wages 1*s*. 6*d*.; *B*'s 2*s*. 1*d*. 7. 4, 6, 8.

1820. 1. 8. 2. $x = \pm 6,\ y = \pm 1,\ z = \pm 3$.
3. $\sqrt{x} = 2,\ -\frac{1}{5}$.
4. $x = -16,\ 8,\ -12;\ y = -1,\ 2,\ -\frac{4}{3}$.
5. 30 sheep, 10 oxen; price of an ox £5; price of sheep £1. 13*s*. 4*d*.
6. 345 to radix 6.

7. Cambridge to Royston 20 miles; Royston to London 36 miles; time of journey 7 and $9\frac{1}{3}$ hours respectively.

1821. 1. 19. 2. $x = -\frac{7}{2}, y = 4$. 3. 14.

4. $x = \frac{7}{20}\cdot\frac{d^2}{b}, -\frac{d^2}{6b}, -\frac{5d^2}{b}$; $y = \frac{4}{15}\frac{d^2}{a}, \frac{11}{18}\frac{d^2}{a}, -\frac{d^2}{a}$.

5. 10 outside places and 18*s*. fare inside.

6. Sum subscribed £8400; price of building £2800; price of instruments £4900.

7. The rates 9, 3; distance 30 miles.

1822. 1. 4. 2. $x = 9, y = 7$. 3. $-a, \frac{a}{2}$.

4. $x = 27, -8, \frac{27}{125}, \frac{8}{125}, -(\frac{19}{12})^3, (\frac{7}{12})^3$.
$y = 8, -27, -\frac{8}{125}, -\frac{27}{125}, (\frac{19}{12})^3, -(\frac{7}{12})^3$.

5. £2250. 6. £144, £48, £16 respectively.

7. £100,000 the sum laid out; £39,062 10*s*. laid out in 5's; £60,937 10*s*. laid out in 3's. Interest in 5's, 4's, 3's, £$4\frac{3}{4}$, £$4\frac{1}{2}$, £$4\frac{1}{4}$ respectively.

1823. 1. 6. 2. $x = 1, y = 2, z = 4$.

3. $\left\{a^{-\frac{1}{2}} \pm b^{-\frac{1}{2}}\right\}^{\frac{4mn}{m-n}}$.

4. $x = 2, -1, \pm\frac{1}{2\sqrt{2}}\{\sqrt{-1} \pm \sqrt{3}\}$; $y = 1, 1, \pm\sqrt{-\frac{1}{2}}$.

5. 1 mile.

6. Amount of debt £1071 17*s*. 6*d*.; price of bonds at beginning of year $81\frac{9}{26}$; rate of interest $7\frac{199}{343}$.

7. $\frac{c}{1-r}\left\{1 + \frac{n(n-1)(1-r)^2\{p(1-r)-1\}}{2r\{n(1-r)-(1-r^n)\}}\right\}$.

1824. 1. 8. 2. $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{4}$. 3. $4, \frac{196}{841}$.

4. $x = 2, \frac{1}{18}, \frac{1}{32}, \frac{1}{18}$; $y = 1, \frac{1}{21}, \frac{1}{28}, \frac{541 \pm \sqrt{1081}}{8100}$.

5. 432.

6. Distance 56 miles; rates of A and B 7 and $9\frac{1}{3}$ miles.

7. The debts £20, £28, £36, £44, £52, £60; value of effects £96.

1825. 1. 4. 2. $x = \pm\frac{9}{2}, \pm\frac{7}{2}$; $y = \pm\frac{7}{2}, \pm\frac{9}{2}$.

3. $\left(\dfrac{a \pm b}{a \mp b}\right)^{\frac{2pq}{q-p}}$. 4. $x = 2, y = 2$.

5. Expenditure and produce first year £60, £90; ditto second year £55, £120.

6. Distance 60 miles; rates of B and C 5 and 10 miles an hour respectively.

7. Wine : spirits in $P = 33:5$; in $Q = 3:35$. Quantity pumped out by A : that by $B = 4:5$.

1826. 1. $\frac{27}{8}$. 2. $x = 3, y = 2$. 3. $\left(\dfrac{a^{2b} - 1}{a^{2b} + 1}\right)^{\pm 2}$.

4. $x = 5, y = 3$. 5. £2000. 6. 5 guineas.

7. 6 miles an hour; distance 60 miles.

1827. 1. 11. 2. $x = 16, y = 25$. 3. $3, \frac{3}{16}$.

4. $x = 6, 4$; $y = 0, 2$. 5. 72 in greater, 12 in less.

6. 6 miles an hour. 7. 432.

1828. 1. $\frac{3}{2}$.

2. $x = \pm\sqrt{ac}, \frac{1}{2}\left(a + c - b \pm \sqrt{(a + c - b)^2 - 4ac}\right)$.
$y = \pm\sqrt{bc}, \frac{1}{2}\left(b + c - a \pm \sqrt{(b + c - a)^2 - 4bc}\right)$.

3. $\pm\frac{1}{2}$.

4. $x = \pm\frac{8}{27}, \pm\frac{1}{8}$; $y = \mp\frac{1}{8}, \mp\frac{8}{27}$.

$$x = \left(\frac{\pm\sqrt{-335} \pm \sqrt{385}}{12}\right)^3, \ y = \left(\frac{\pm\sqrt{-335} \mp \sqrt{385}}{12}\right)^3;$$

$$x = \left(\frac{\pm\sqrt{95} \pm \sqrt{5}}{12\sqrt{2}}\right), \ y = \left(\frac{\pm\sqrt{95} \mp \sqrt{5}}{12\sqrt{2}}\right)^3.$$

5. 11 o'clock. 6. The tack is that corresponding to the whole no. equal to or next greater than $\dfrac{2}{p}\left(q - \dfrac{r}{3}\right)$.

7. Time in second 2 hours; breadth of each level 10 feet; length of levels at first 100, and at second irruption 200 feet.

1829. 1. 4. 2. $x = 5, \frac{1}{29}(15 \pm 6\sqrt{-1})$;
$y = 3, \frac{1}{29}(25 \pm 10\sqrt{-1})$.

3. $\left(\dfrac{a^{\frac{r}{s}} + 1}{a^{\frac{r}{s}} - 1}\right)^{\frac{1}{(m+n)^2}}$.

4. $x = 4, -2(1 \mp \sqrt{-3})$; $y = \frac{1}{4}, -\frac{1}{8}(13 \mp 3\sqrt{-3})$.

5. 150. 6. From A to B is 10 miles; from B to C 24 miles; from A to C 26 miles; rates are 3 and 9 miles per hour respectively. 7. £27.

1830. 1. 4. 2. $x = 4, \frac{4}{17}$; $y = 12, \frac{36}{19}$.

3. $1, 16, \dfrac{1 \pm 3\sqrt{-7}}{2}$.

4. $x = 1, (-1 \pm \sqrt{-2})^2$; $y = 4, -2$. 5. 3 miles.

6. Gross revenue and interest 64 and 28 millions.

7. 24 : 21 : 22.

1831. 1. $x = 21, y = 20$. 2. $\sqrt[3]{\dfrac{9a^2b - 4b^3}{4}}$.

3. $\pm \dfrac{1}{a}\left\{-1 \pm \sqrt{1 - a^4} \pm \sqrt{2(1 \mp \sqrt{1 - a^4})}\right\}^{\frac{1}{2}}$.

4. $x = \pm \frac{1}{2}(4 \pm \sqrt{6} \pm \sqrt{18 \pm 8\sqrt{6}})$;
$y = \pm \frac{1}{2}(2 \pm \sqrt{6} \pm \sqrt{18 \pm 8\sqrt{6}})$.

5. £20. 6. 2 : 3. 7. 20 O.; 40 S.; 400 P.

1832. 1. $-\frac{1}{2}, \frac{8}{11}$.

2. $x = (\sqrt{2} + 1)^2, (\sqrt{2} - 1)^2$; $y = 1, (\sqrt{2} - 1)^4$.

3. $\dfrac{a}{2}, \dfrac{a}{6}(-5 \pm \sqrt{37})$. 4. $x = 4, \frac{1}{16}$; $y = 9, \frac{9}{64}$.

5. £240; 120 acres.

6. Distance $54\frac{5}{6}$ miles; rate of Times and Fly $8\frac{3}{4}$, and 8 miles an hour.

7. Votes for A, 150; B, 140; C, 170.

1833. 1. 6. 2. $x = 4, y = 3$.

3. $9 \pm 4\sqrt{7}, \frac{1}{2}(3 \pm \sqrt{13})$.

4. $x = \sqrt{\dfrac{m^2 - 1}{m^2 + 1}},\ y = \sqrt{\dfrac{n^2 - 1}{n^2 + 1}}.$

where $n^2 = \dfrac{1 \pm \sqrt{3} \pm \sqrt{\pm 2\sqrt{3}}}{2},\ m^2 = \sqrt{\dfrac{n^4 + 1}{n^4 - 1}}.$

5. A began at $4\frac{1}{2}$ A.M., B at 5 A.M.

6. 90, 120, 150 yards.

7. £18, £32.

1834. 1. $\frac{14}{13}$.

2. $x = b^{\frac{2n}{m+n}} \left\{ a^{\frac{m-n}{2}} \pm \sqrt{a^{m-n} - b^{m-n}} \right\}^{\frac{2}{m+n}}$

$y = a b^{\frac{m-n}{m+n}} \left\{ a^{\frac{m-n}{2}} \pm \sqrt{a^{m-n} - b^{m-n}} \right\}^{-\frac{2}{m+n}}.$

3. $4 \pm \sqrt{6},\ \pm \sqrt{-2}.$

4. $x = y = \pm a\sqrt{2}.$

5. 1080 yards; $16\frac{1}{2}$ minutes.

6. 12 in large wheel, 8 in small.

7. £30600.

1835. 1. 3.

2. $x = \frac{3}{2},\ y = \frac{4}{3},\ z = \frac{5}{4}.$

3. $-1,\ \frac{1}{2}\{p \pm \sqrt{p^2 - 4}\}$

where $p = \dfrac{-(4a+1) \pm \sqrt{5(4a+1)}}{2(a-1)}.$

4. $x = \dfrac{a-1}{b},\ y = \{1 + \sqrt[3]{2a - 1}\}^3$

5. AD, AB, CB, 290, 230, 200 yards respectively.

6. If a, b, c = no. of acres A, B, C reap per day, no. of acres in field $= (a + b + c)\left\{p + \dfrac{m(a+b)(pc-1)}{qac}\right\}$ $= k$ suppose; deduction made $= \dfrac{q}{pc(pc-1)}$ shillings per acre, sum A received $= \dfrac{a(k - pc)q}{(a+b)(pc-1)}$ shillings, sum B received $= \dfrac{b(k - pc)q}{(a+b)(pc-1)}$ shillings.

7. Plumpers for $P = 4$; $Q = 558$; $R = 3$. Final state of poll P, 693; Q, 688; R, 736.

1836. 1. $\frac{12}{5}$.

2. $x = \pm \frac{3}{2}, \pm \frac{3}{2}\sqrt{-1}$; $y = \pm \frac{1}{2}, \mp \frac{1}{2}\sqrt{-1}$.

3. $\frac{2}{3}, \pm \frac{4}{3}\sqrt{2}, \frac{4}{9}(-2 \pm \sqrt{-14})$.

4. $x = 0, 1, y = \pm \sqrt{a}$.

$x = \frac{1}{2}(1 \pm \sqrt{1 + 4p}), y = \pm \sqrt{\frac{a}{b}(b - p)}$.

where $p = \frac{(2a^2 - (a - 1)b)b}{a^2 - ab + b^2}$.

5. $10\frac{1}{2}$ minutes. 6. 2*s*. 1*d*.

7. 6325 gallons, 4650 gallons.

1837. 1. 3.

2. $x = 1 + a, y = \frac{1}{2}\{\sqrt{1 - a + a^2} + \frac{1}{2} - a\}$;

$z = \frac{1}{2}\{\sqrt{1 - a + a^2} - \frac{1}{2} + a\}$

3. $4\sqrt{2}$.

4. $x = \frac{1}{2}\left\{\sqrt{\sqrt[3]{3} + 3} + \sqrt{\sqrt[3]{3} - 1}\right\}$.

$y = \frac{1}{2}\left\{\sqrt[3]{3} \,.\, \sqrt{\sqrt[3]{3} + 3} \pm \sqrt{3\sqrt[3]{9} - 1}\right\}$.

5. Money £672; debts £840. 6. 15 minutes.

7. 10.

1838. 1. $-2, \frac{1}{3}$.

2. $x = \pm 1, 2, -2\sqrt[3]{4}$; $y = \pm 1, 2, -\sqrt[3]{2}$; $z = 4, 1, 1$.

3. $1, -3, -\frac{1}{2}$.

4. $x = 0, \pm \frac{\pm\sqrt{3} - 1}{\sqrt{11 \mp 6\sqrt{3}}}$; $y = 0, \pm \frac{1}{\sqrt{11 \mp 6\sqrt{3}}}$.

5. 1600. 6. 16 passengers originally, fare 3*d*.

7. 16 miles.

1839. 1. -5.

2. $x = 2(a \pm b + \sqrt{a^2 \pm ab + b^2}), y = 2(b \pm a \mp \sqrt{a^2 \pm ab + b^2})$.

3. $2401, \left(\frac{-7 \pm \sqrt{37}}{2}\right)^4, \left(\frac{7 \pm \sqrt{37}}{2}\right)^4$.

4. $x = \frac{3 \pm \sqrt{3}}{2}, \frac{1 \pm \sqrt{3}}{2}, \frac{3 \pm \sqrt{-29}}{2}, \frac{1 \pm \sqrt{-13}}{2}.$

$y = \frac{3 \mp \sqrt{3}}{2}, \frac{1 \mp \sqrt{3}}{2}, \frac{3 \mp \sqrt{-29}}{2}, \frac{1 \mp \sqrt{-13}}{2}.$

5. $x = y = a - b.$ $\qquad x = a - \frac{b}{2}\left\{ m \pm \sqrt{m^2 - 4}\right\};$

$y = \frac{1}{2}\left\{a - \frac{b}{2}(m \pm \sqrt{m^2 - 4})\right\}\left\{m \pm \sqrt{m^2 - 4}\right\}.$

where $m = \frac{1}{2b}(a - b \pm \sqrt{a^2 + 2ab + 5b^2}).$

6. $2m - n + \frac{8p}{q} \cdot \frac{a+d}{a-e} \cdot \frac{a+e-c}{a+d+c}$ no. of passengers;

$(2m - n)\frac{q}{4} \cdot \frac{a-e}{a+b} \cdot \frac{a+b+c}{a-e+c} + p \cdot \frac{a+d}{a+d+c}$ no. of galls.

7. Material 40 francs, $4\frac{4}{9}$ francs respectively; workmanship 20 francs, 60 francs.

8. The n^{th} term of first row $= n^2$; n^{th} term of seventh row $= n^2 + \frac{77}{12}n + \frac{419}{48}$.

1840. 1. $4\frac{1}{3}$.

2. $x = \frac{3}{2}(19 \pm \sqrt{105}), y = \frac{1}{6}(3 \pm \sqrt{105}).$

3. $4, \frac{25}{4}, \left(\frac{\pm\sqrt{249} - 13}{4}\right)^2.$

4. $x = \left\{\frac{16}{183}(\pm\sqrt{757} - 5)\right\}^{\frac{4}{3}}, \left\{\frac{16}{183}(7 \pm \sqrt{781})\right\}^{\frac{4}{3}};$

$y = \frac{8}{183}(\pm\sqrt{757} - 5), \frac{8}{183}(7 \pm \sqrt{781}).$

5. Cost price per bushel £$\frac{4}{31}$; 50 per cent.

6. A's gain in min. per hour $= \frac{3.24.60.\, a - a^2}{24\,(2.24.60 - a)}$;

B's $= 120 \cdot \frac{3.24.60.\, a - a^2 - 24^2.\, 60^2}{(2.24.60 - a)^2}.$

7. A's rate $= \frac{n}{r} \cdot \frac{bs + ct}{m + p - q} - \frac{ct}{r}$;

height of hill $= \frac{1}{r^2}\left\{nbs - qct + n \cdot \frac{bs + ct}{m + p - q}\right\}.$

1841. 1. $\pm 9\sqrt{2}$. 2. $x = 1, 9$; $y = 2, -6$; $z = 4, 4$.

3. $\dfrac{5 \pm \sqrt{261}}{2}$, $\dfrac{5 \pm \sqrt{-19}}{2}$.

4. $x^2 = y^2 = \dfrac{b^2 - a^2}{2}$.

$$x^2 = \frac{3b^2 - 11a^2 \pm 2\sqrt{b^4 - 4a^2b^2 - a^4}}{10};$$

$$y^2 = \frac{7b^2 + a^2 \mp 2\sqrt{b^4 - 4a^2b^2 - a^4}}{10}.$$

5. 1, 3, 4 feet.

6. Votes for successful C; 34 from A, 482 from B. For unsuccessful C; 73 from A, 68 from B.

7. 100 yards per minute; distance 9845 yards.

1842. 1. $\frac{65}{66}$. 2. $x = 3, y = -\frac{5}{2}$.

3. $3, -\frac{1}{3}, \frac{1}{3}(4 \pm \sqrt{13})$.

4. $x = \pm\frac{1}{2}\sqrt{5}, 0$; $y = \pm\frac{1}{2}\sqrt{6}, \pm\sqrt{-1}$.

$x = \pm\frac{1}{2}\sqrt{\frac{1}{2}(1 \pm \sqrt{33})}$, $y = \pm\frac{1}{2}\sqrt{5 \pm \sqrt{33}}$.

5. 84 miles. 6. $\frac{3}{2}$, $\frac{1}{4}$ inches.

7. 60 qrs. of wheat; ratio $= 44 : 35$.

1843. 1. $\pm 1, \pm 8$. 2. $x = 6, \frac{10}{3}$; $y = 5, \frac{19}{3}$; $z = 3, \frac{13}{3}$;

$$x = \frac{-14 \mp \sqrt{-2516}}{3},\ y = \frac{73 \pm \sqrt{-2516}}{3},$$

$$z = \frac{-101 \pm \sqrt{-2516}}{3}.$$

3. $\pm\dfrac{a}{2} \cdot \dfrac{\sqrt{1 + 2n} \pm \sqrt{1 - 2n}}{\sqrt{n}}$

$\pm\dfrac{a}{2} \cdot \dfrac{\sqrt{5n - 3} \pm \sqrt{n + 1}}{\sqrt{n - 1}}$.

4. $x^3 = p^2 . (c \pm \sqrt{c^2 - p^2})$; $y^3 = p^2 . (c \mp \sqrt{c^2 - p^2})$ where $p^2 = (a^2 - 2bc - 2b^2)$.

5. Between P and T he spent 105 rubles; between T and M he spent 90 notes or 108 rubles.

6. 11 oz. of gold, 18500 of silver, 1789 of copper.

7. 12 fortresses, 504 men.

1844. 1. $\frac{5}{2}$. 2. $3, -1, -1 \pm \sqrt{6}$.

3. $x = 1, y = \frac{3}{2}, z = 2$.

4. $x = \pm 1, \pm \sqrt{\frac{-37}{6}}$; $y = \pm 4, \mp \frac{167}{12}\sqrt{\frac{-6}{37}}$;

$$x = \pm\frac{1}{2}\sqrt{\frac{3}{2}\left(\pm\sqrt{129}-7\right)},\ y = \pm\frac{1}{4}\sqrt{\frac{\pm 9\sqrt{129}-55}{6\left(\sqrt{129}-7\right)}}.$$

5. 500 pages of MS.; estimate £$44\frac{4}{9}$. 6. 10.

7. They worked together $1\frac{1}{4}$ hours; whole time $13\frac{1}{8}$ hours.

1845. 1. 2, 3. 2. $49, 64, \frac{1}{2}(93 \pm \sqrt{185})$.

3. $\pm\frac{7}{6}, \pm\frac{14}{13}, -\frac{3}{2}, \pm\frac{3}{2}\sqrt{-1}$.

4. $x = -\frac{a}{2}, \pm a\sqrt{-2}$; $y = -\frac{a}{2}, \pm a\sqrt{-\frac{1}{2}}$.

5. £400. 6. 41 cubic inches. 7. 20 miles.

1846. 1. ± 5. 2. $\left(\frac{\sqrt{21}+1}{2}\right)^2, \left(\frac{\sqrt{-3}-1}{2}\right)^2$.

3. $2(1+\sqrt{2}), 1-\sqrt{2} \pm \sqrt{3-4\sqrt{2}}$.

4. $x = y = 0$; $x = y = a^2 + b^2$.

$$x = -\frac{a^2+b^2}{2}\cdot\left(\frac{1}{a+b}+\frac{p-1}{p+1}\right),$$

$$y = -\frac{a^2+b^2}{2}\cdot\left(\frac{1}{a+b}-\frac{p-1}{p+1}\right) \text{ where } p = \frac{1\pm\sqrt{-3}}{2}.$$

5. 2170. 6. No. allotted 4000, reserved 6000.

7. No. of factories 5. Tax on 1st £195; on last £135.

1847. 1. 78.

2. $2v = \pm\sqrt{a+b+c} \pm \sqrt{2b} \pm \sqrt{2a} \pm \sqrt{2c}$;

$$2x = \pm\sqrt{a+b+c} \mp \sqrt{2b} \mp \sqrt{2c} \pm \sqrt{2a};$$

$$2y = \pm\sqrt{a+b+c} \pm \sqrt{2b} \mp \sqrt{2a} \mp \sqrt{2c};$$

$$2z = \pm\sqrt{a+b+c} \mp \sqrt{2b} \mp \sqrt{2c} \pm \sqrt{2a}.$$

3. $x = y = 2$. 4. $\dfrac{\pm\sqrt{a+1}-1}{\pm\sqrt{a+1}+1}$.

5. 3 yards. 6. *A*'s, 6 or 5; *B*'s, 5 or 4 miles per hour.

7. Johnian voters for L. P. 318; for P. A. 53. Trinity voters for L. P. 202; for P. A. 378.

1848. 1. $\frac{411}{662}$. 2. $\pm\sqrt{a^2-(a-b)^2}$; $\pm\sqrt{a^2-(a+3b)^2}$.

3. $x = 27, 15, 30, \frac{7}{2}(3\pm\sqrt{33})$; $y = 27, 30, 15, \frac{7}{2}(3\mp\sqrt{33})$.

4. $x = \dfrac{a}{2}(1\pm\sqrt{3})$; $y = \dfrac{a}{2}\left(1\pm\dfrac{1}{\sqrt{3}}\right)\sqrt{1\pm\dfrac{4}{\sqrt{3}}}$.

5. $AB = 31\frac{1}{2}$ miles; $BC = 63$.

6. $\dfrac{m'}{(a-a')+n(c'-c)}$, $\dfrac{m'n}{(a-a')+n(c'-c)}$. 7. 100.

1849. 1. $\dfrac{a^2+4(b-c)^4}{4(b-c)^2}$. 2. 1, 1.

3. $\left(\dfrac{3}{2}\right)^4, (3)^4, \left(-\dfrac{9}{4}\right)^4$.

4. $x = \frac{1}{16}(\pm\sqrt{1\pm 36\sqrt{2}}-1)^2$, $\frac{9}{16}(1\pm\sqrt{1\pm 4\sqrt{2}})^2$;
$y = \frac{1}{8}(\pm\sqrt{1\pm 36\sqrt{2}}-1)^2$, $\frac{9}{8}(1\pm\sqrt{1\pm 4\sqrt{2}})^2$.

5. Arrival at *B, C, D, E*; 9^h, $10^h.\ 3^m$, $11^h.\ 6^m$, $12^h.\ 9^m$.

6. $r = n = 31$. 7. $\frac{1}{20}, \frac{1}{30}, \frac{1}{40}$.

1850. 1. $\frac{1}{2}(\pm\sqrt{2-a}+\sqrt{a})^2$, $1\pm\sqrt{2a-a^2}$.

2. $\dfrac{1}{2}\left\{1\pm\sqrt{\dfrac{1\pm\sqrt{33}}{8}}\right\}$.

3. $x = (1\pm\sqrt{5})\dfrac{c}{2}$, $y = (-1\pm\sqrt{5})\dfrac{c}{2}$.

4. $x = \dfrac{\pm a\pm b+\sqrt[3]{m^3-c^3}\,(1-\sqrt{3})}{1+\sqrt{3}}$,

$y = m+\sqrt[3]{m^3-c^3}$,

$z = m-\sqrt[3]{m^3-c^3}$. where $m = \dfrac{\pm a\mp b}{\sqrt{3}-1}$.

5. £320.

6. P's velocity 12 miles, Q's 10 miles per hour.
If P describe ABC they meet at C.
If Q describe ABC they meet in CB at a distance of $\frac{16}{3}$ miles from C.

7. 1957.

1851. 1. $0, -\frac{5}{2}$. 2. $3 \pm \sqrt{3}$. 3. $\dfrac{1+\sqrt{2} \pm \sqrt{8\sqrt{2}-7}}{\sqrt{2}}$.

4. $x = \frac{3}{2}, \frac{1}{2}$; $y = \frac{1}{2}, \frac{3}{2}$; $z = 3, 3$.

5. A's income £1000; B's, £600; C's, £700.

6. 116 yards.

7. Rates of A, B, C, 3, 4, 5 miles per hour.

J. WEBB, PRINTER, CAMBRIDGE.